

**Work = Force x displacement**

$$= (F) (s)$$

N

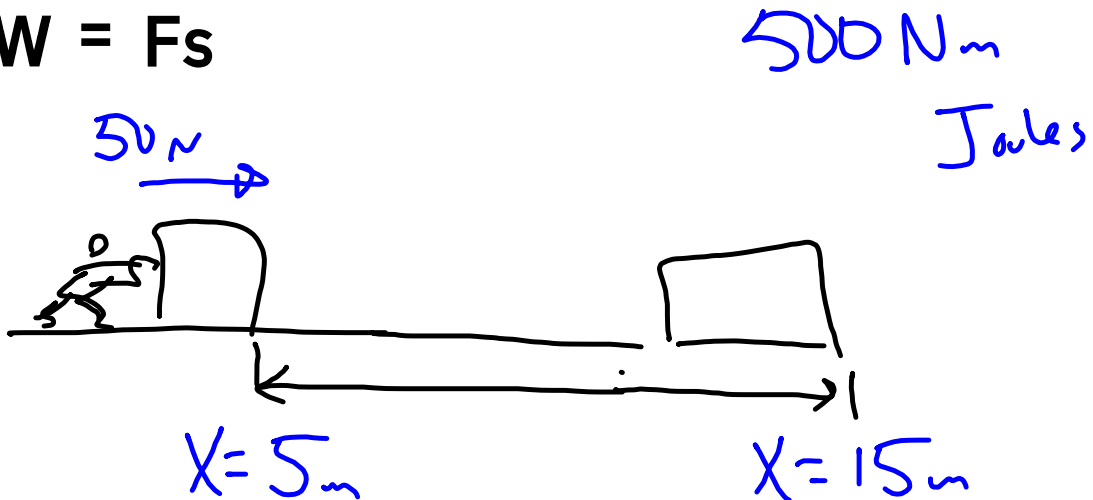
m

.

# 1. How do you calculate Work?

Work = Force x Displacement

$$W = Fs$$



If the box moves upward at a constant ~~2 m/s~~, how much work was done by the Tension in the rope?

Diagram: A box labeled "100 kg" is on a surface. A rope is attached to it and extends upwards to a point 10 m above the surface. A second box is shown suspended from the rope at this height.

Free-body diagram: A central point has an upward arrow labeled  $T = 10,000 \text{ N}$  and a downward arrow labeled  $W = 1000 \text{ N}$ . Below the diagram is the text  $a = ?$ .

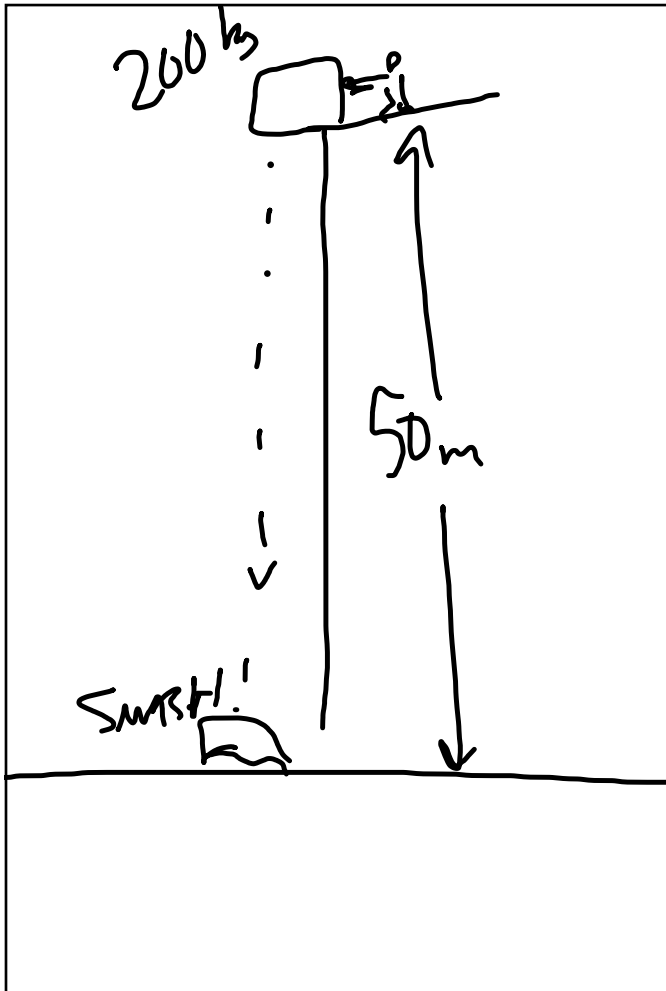
Handwritten calculations:

$$W = F_s$$

$$= T(\Delta y)$$

$$= (1000 \text{ N})(10 \text{ m})$$

$$= 10,000 \text{ J}$$



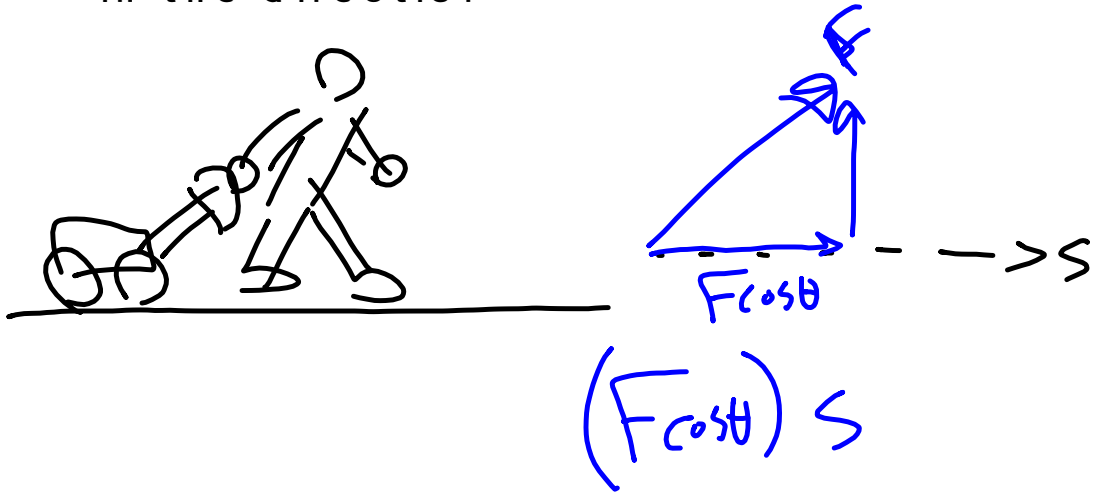
The crate topples from rest to smash onto the ground. Find the work done by gravity.

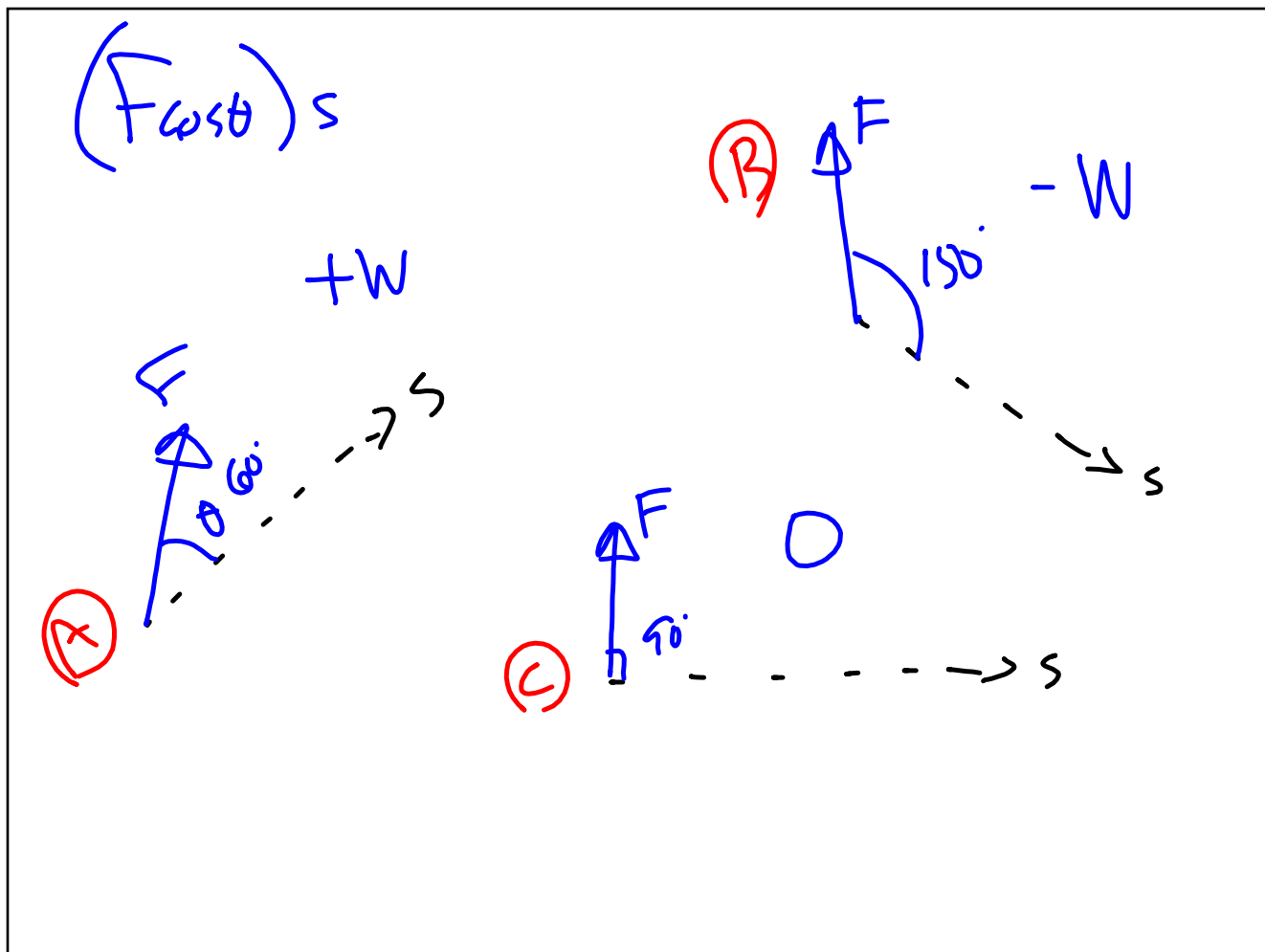
$$\begin{aligned} W &= F s \quad mg \downarrow \\ &= mg(\Delta y) \\ &= (200)(10)(50) \\ &= 100,000 \text{ J} \end{aligned}$$

A small diagram to the right of the equations shows a dashed vertical line with a downward arrow labeled  $\Delta y$ . A solid arrow labeled  $mg$  points downwards from the top of the dashed line.

2. What if there's an angle between the force and displacement?

Work =  $F$   $s_{\text{component}}$   
in the direction





Find the work done by the Tension.

$= \vec{F} \cdot \vec{s} =$

$T \cos \theta$   
 $(100) \cos 45 = 70.7 \text{ N}$   
 $707 \text{ J}$

$100 \text{ N}$   
 $45^\circ$   
 $10 \text{ m}$

$W = |\vec{F}| \cdot |\vec{s}| \cos \theta$

In general...

$$W = F S \cos \theta$$

magnitude      magnitude      angle between  
F and S



## Dot Product

A way of multiplying vectors

The answer is a scalar

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos\theta$$

$$W = \vec{F} \cdot \vec{s} = |\vec{F}| |\vec{s}| \cos\theta$$

What if you're in i j k notation?

(multiply like components)

$$\vec{F} = (3\hat{i} + 2\hat{j} - 5\hat{k}) \text{ N}$$

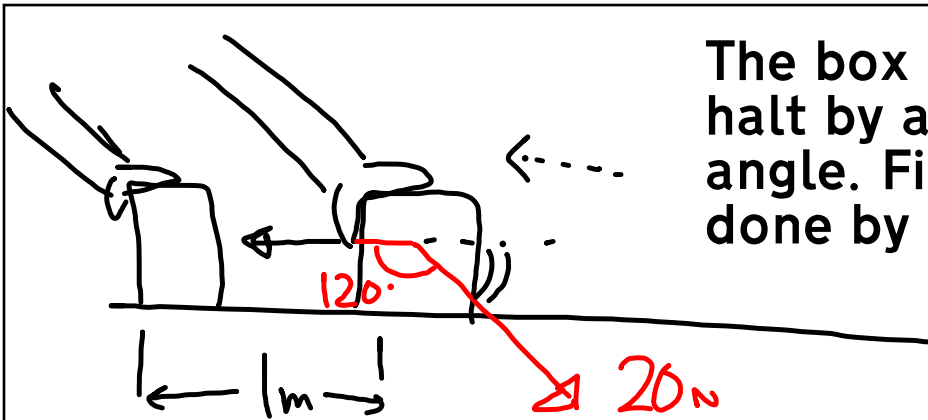
$$\vec{S} = (-2\hat{i} + 5\hat{j} - 6\hat{k}) \text{ m}$$

$$-6 + 10 + 30 = 34 \text{ J}$$

$$\vec{F} = (6\hat{i} - 5\hat{j} + 0\hat{k}) \text{ N}$$

$$\vec{S} = (3\hat{i} + 2\hat{j} + 6\hat{k}) \text{ m}$$

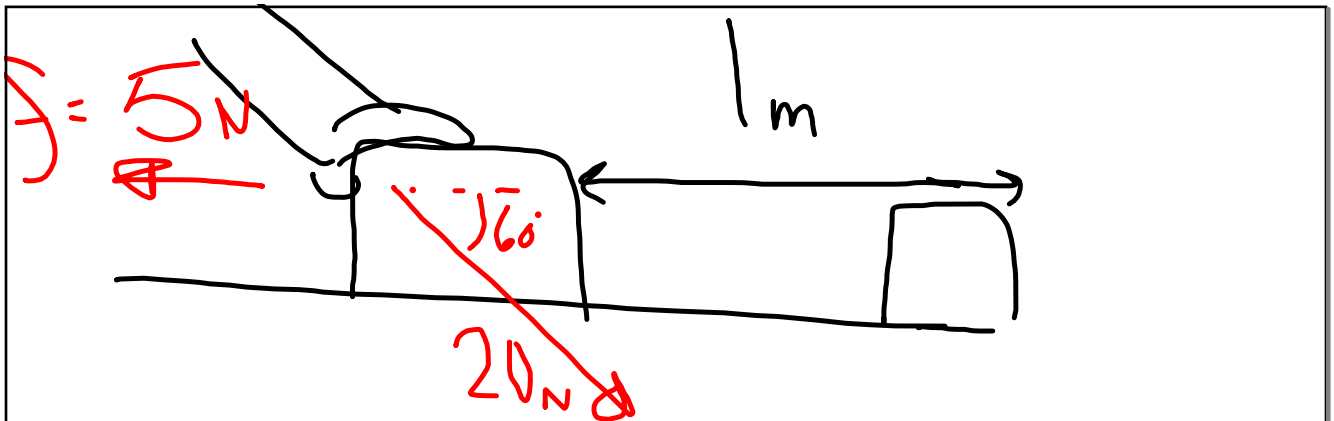
$$18 - 10 \quad 0 = 8 \text{ J}$$



The diagram shows a box on a horizontal surface. A hand is shown pushing the box from the left. A red arrow labeled  $20\text{ N}$  points downwards and to the left, making an angle of  $120^\circ$  with the horizontal surface. A horizontal arrow labeled  $1\text{ m}$  indicates the displacement of the box to the left. A dashed arrow points to the right, indicating the direction of motion before the push.

The box is brought to a halt by a push at an angle. Find the work done by the hand.

$$W = |\vec{F}| \cdot |\vec{s}| \cos\theta$$
$$= (20\text{ N})(1\text{ m}) \cos 120^\circ$$
$$= -10\text{ J}$$



Find the work done by...

a) The push from the hand.

b) Friction.

c) Normal.

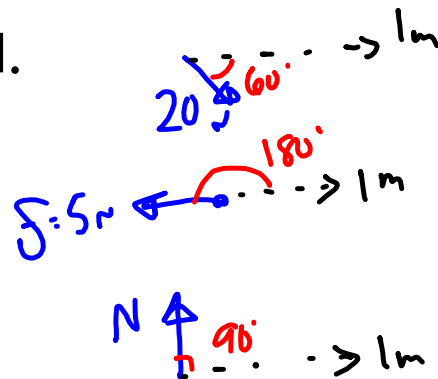
10 J

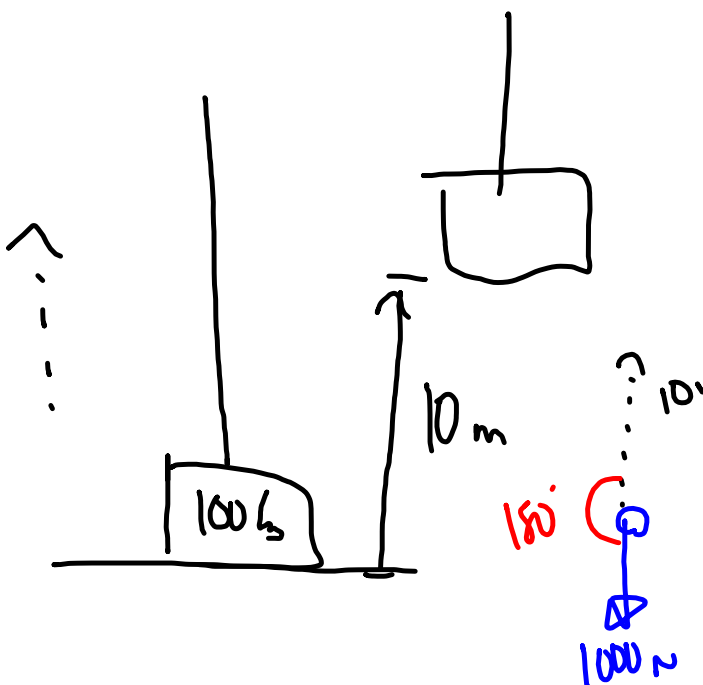
-5 J

0 J

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5 J



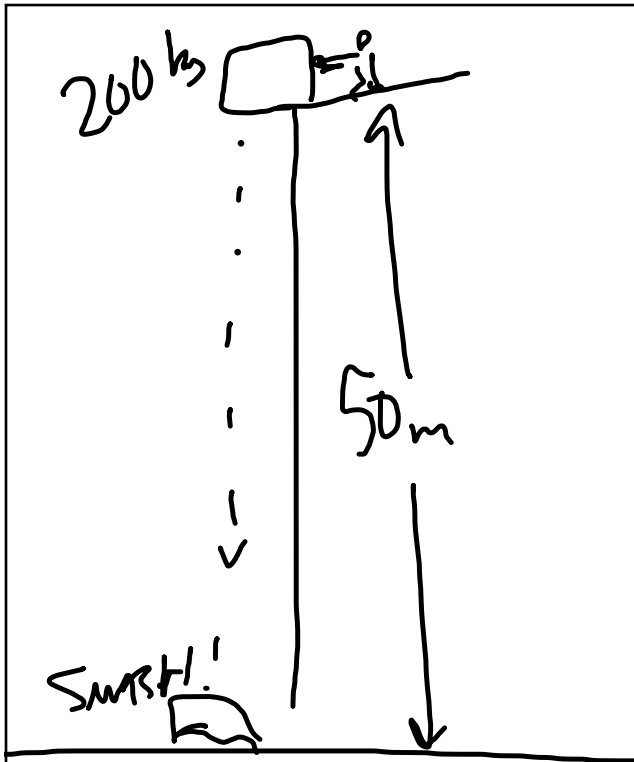


The crate moves upward at a constant 2 m/s. Now find the work done by gravity.

$$W_T = 10,000 \text{ J}$$
$$W_{mg} = -10,000 \text{ J}$$

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$$W_{\text{net}} = 0$$



A hand-drawn diagram illustrating a physics problem. A rectangular block is shown at the top of a vertical dashed line, representing a height of 50 m. The block is labeled "260 kg". An upward-pointing arrow next to the block is labeled "50 m". Below the block, a solid vertical line extends to a horizontal ground line. A downward-pointing arrow next to this line is labeled "50 m". At the point where the block hits the ground, there is a small squiggle and the word "SMASH!" written next to it. The ground is represented by a horizontal line.

How much work was done by the ground?

$$W_{mg} = 100,000 \text{ J}$$
$$W_{\text{ground}} = -100,000 \text{ J}$$

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$$W_{\text{net}} = 0$$

$W_{\text{grav}} = ?$

1.5m

20kg

$W_{\text{mg}} = -300\text{J}$

$W_{\text{pm}} = 300\text{J}$

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0

1.5m

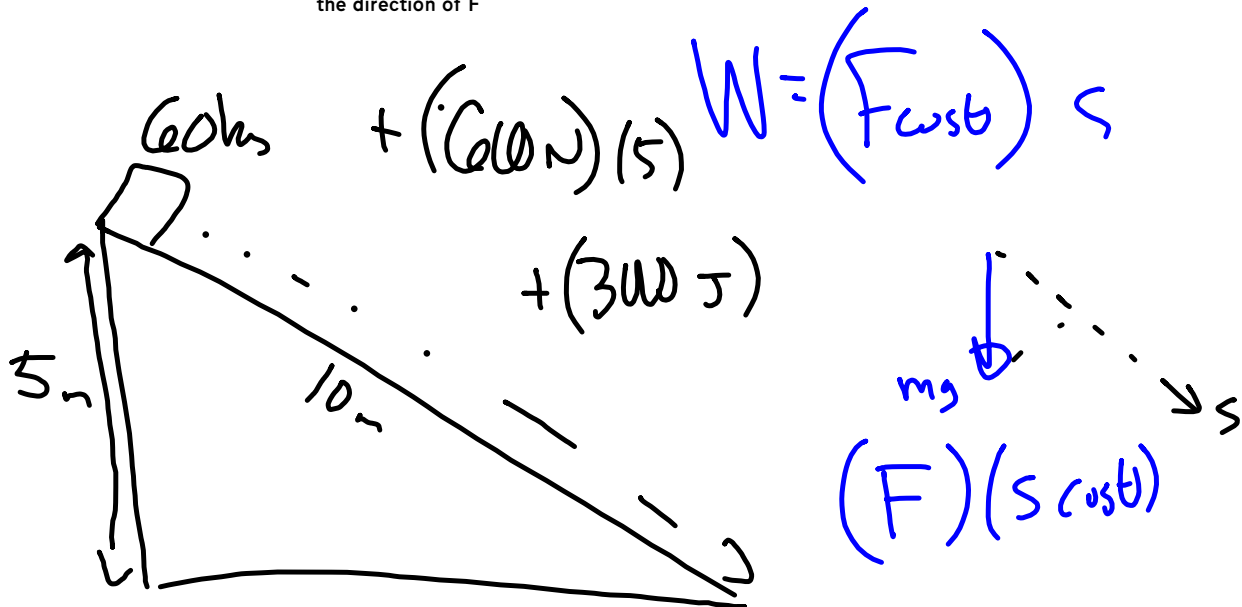
$\theta = ?$

200N



Another way of looking at it...

Work =  $F s_{\text{component in the direction of } F}$



$$W_{\text{net}} = K_f - K_i$$

$$300J = \frac{1}{2} m v_f^2$$

$$300 = \frac{1}{2} (60) v_f^2$$

$$\Delta x = \frac{1}{2}(v_0 + v)t$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

← (a) →

$$\sum_i \vec{F} = m\vec{a}$$

$$W = \vec{F} \cdot \vec{s}$$

$$= |\vec{F}| \cdot |\vec{s}| \cos\theta$$

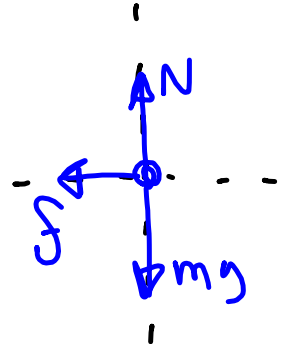
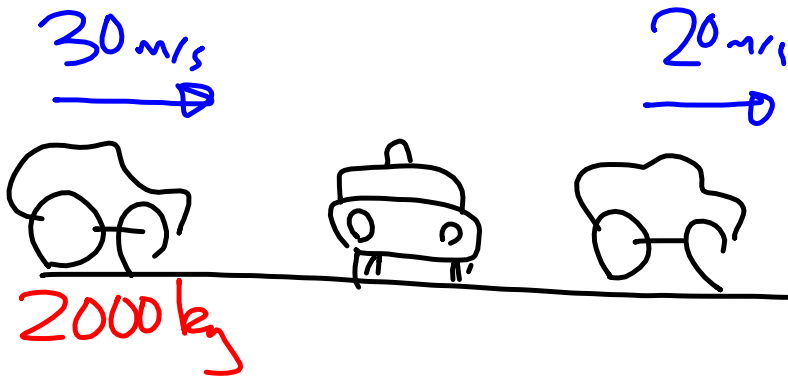
$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

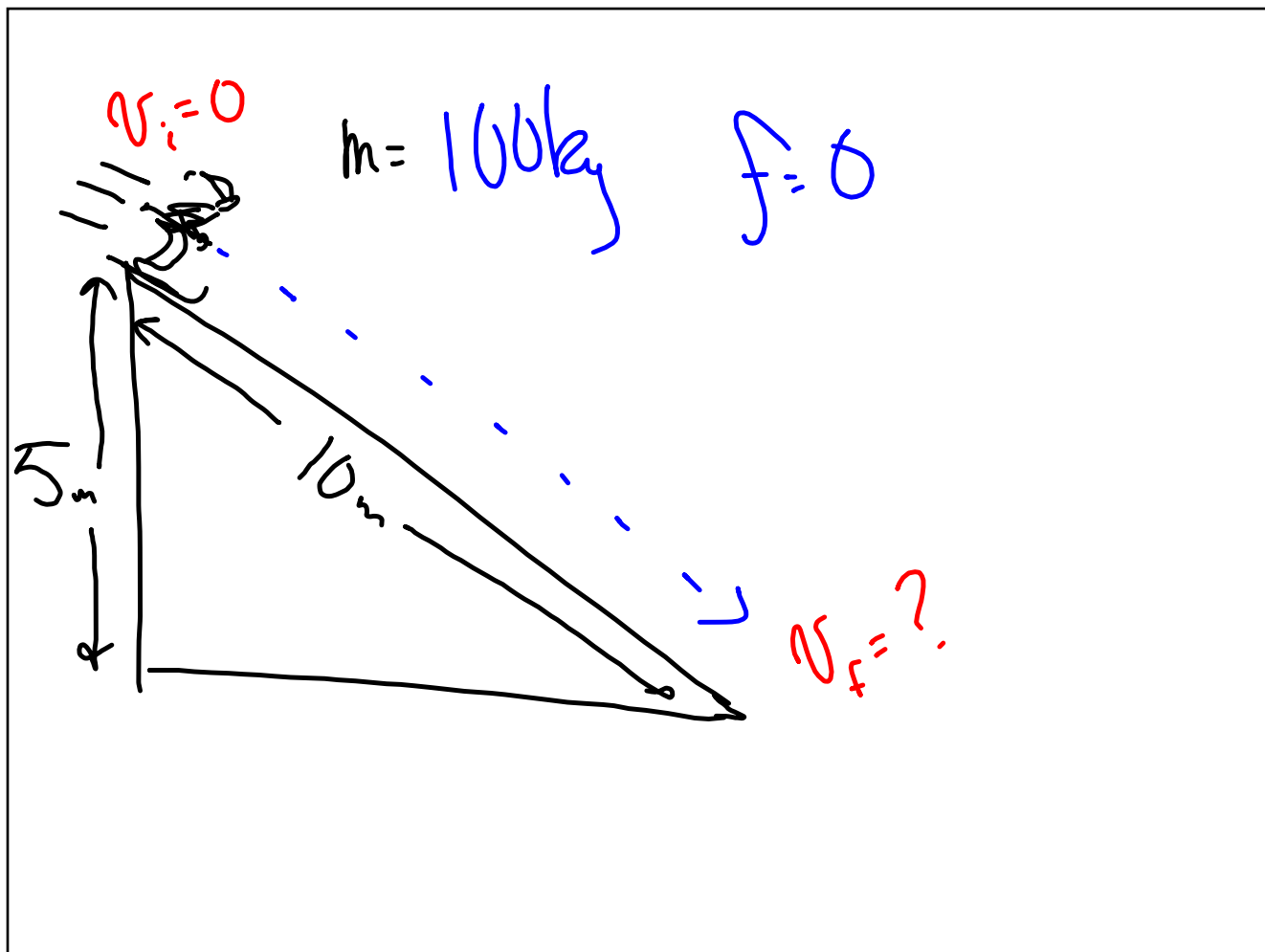
$$W_{\text{total}} = \Delta K$$

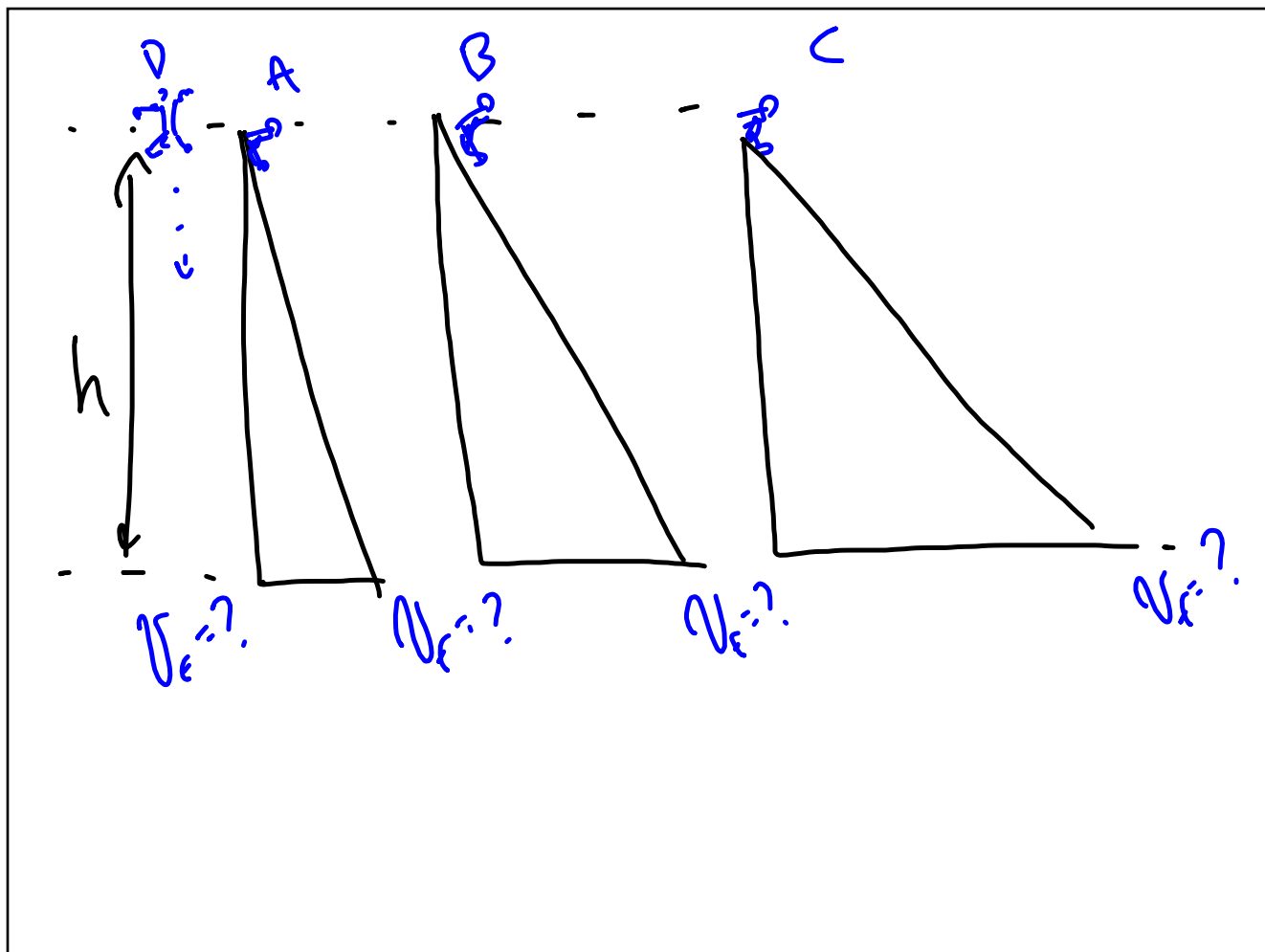
$$\Delta K = W_{\text{total}}$$

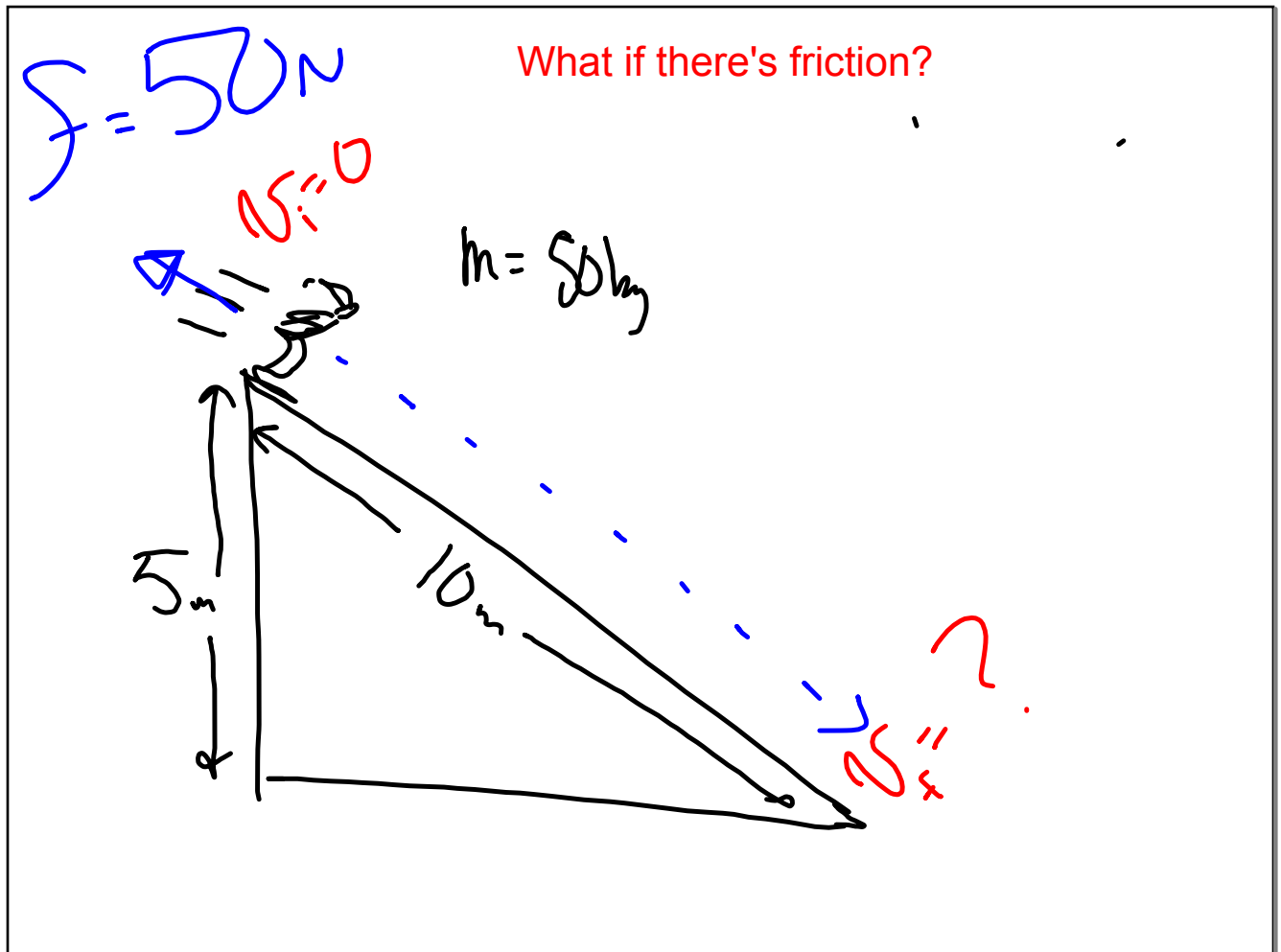
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{\text{total}}$$

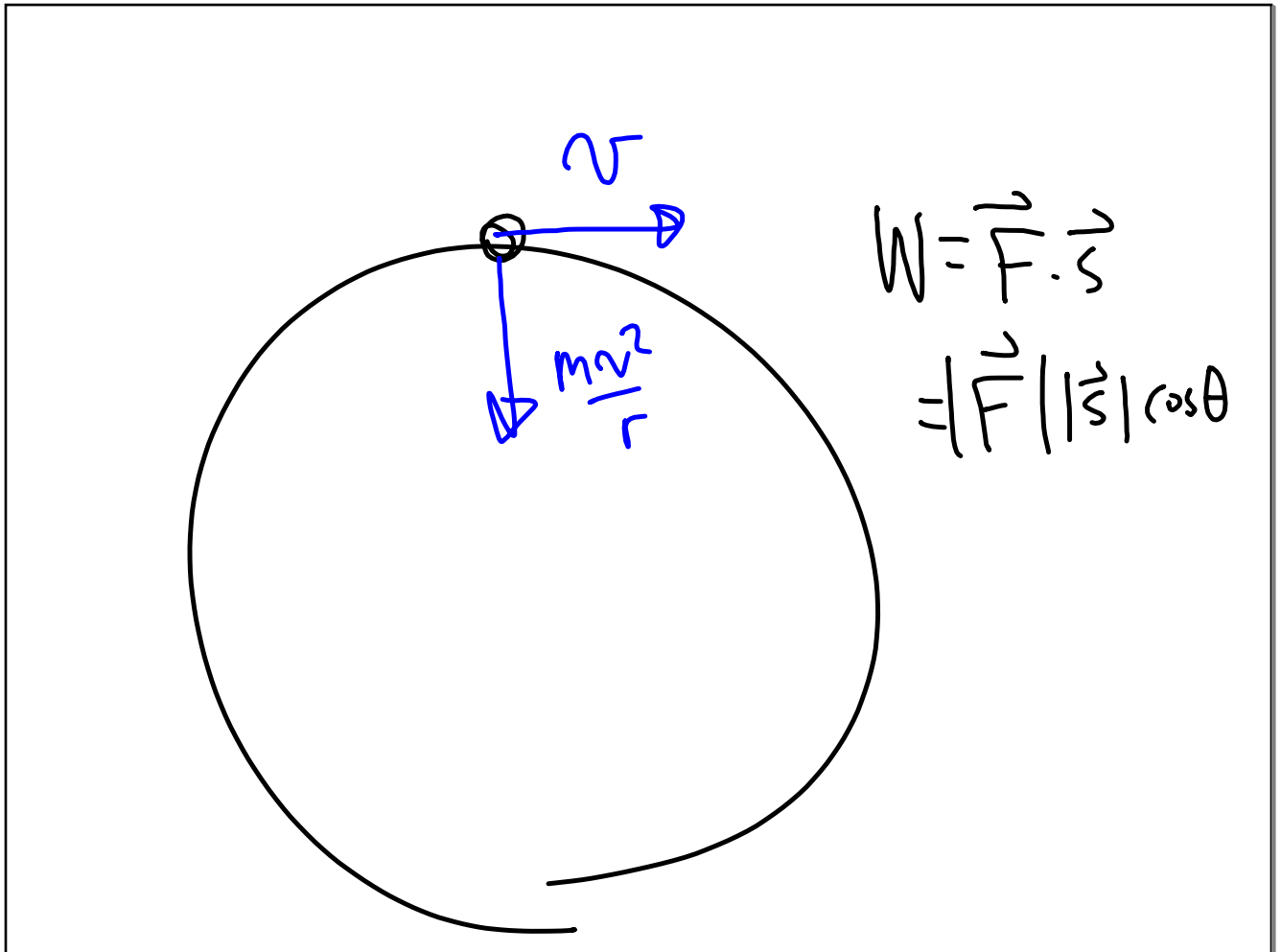
$W_{brakes} = ?$

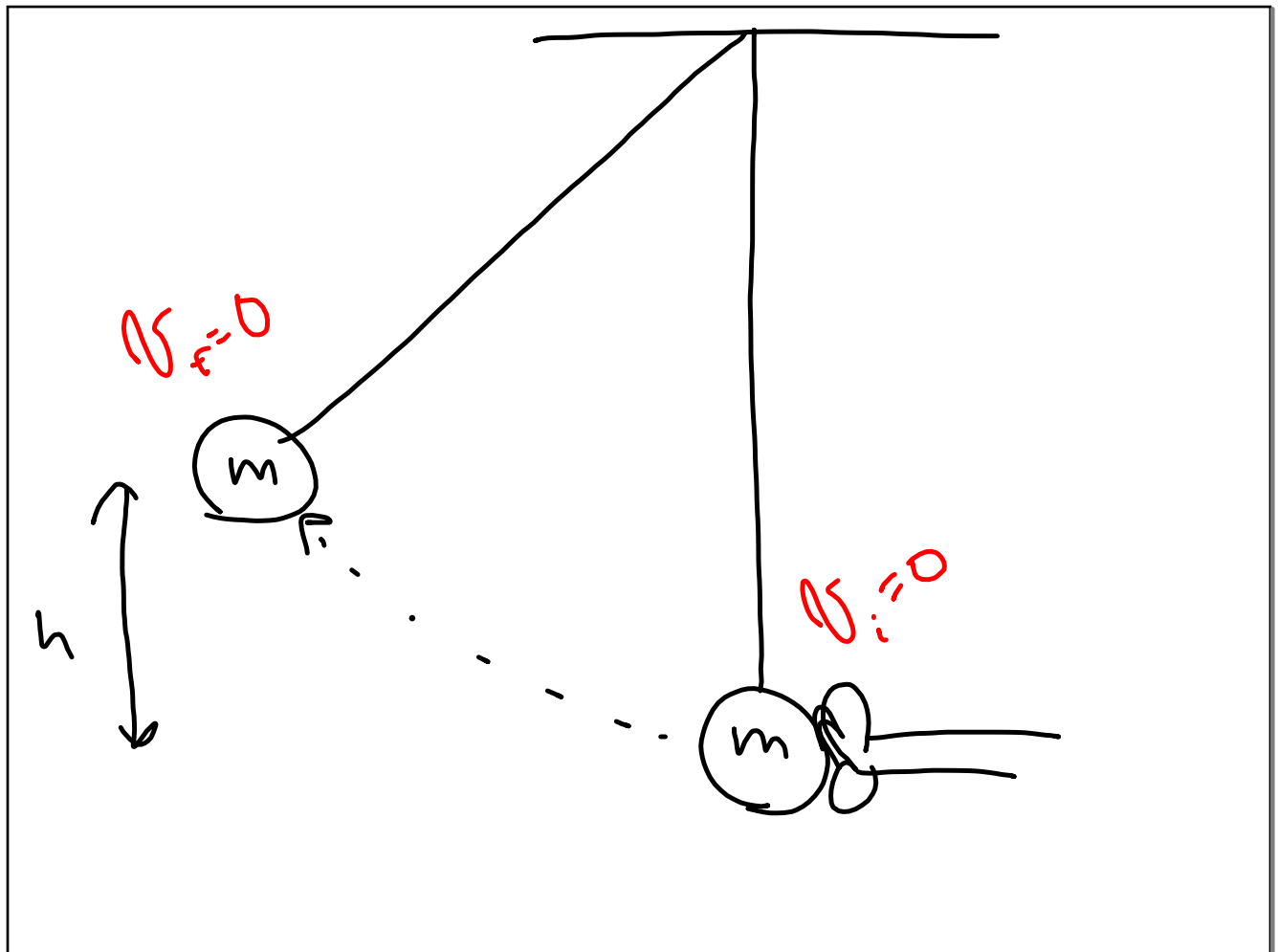




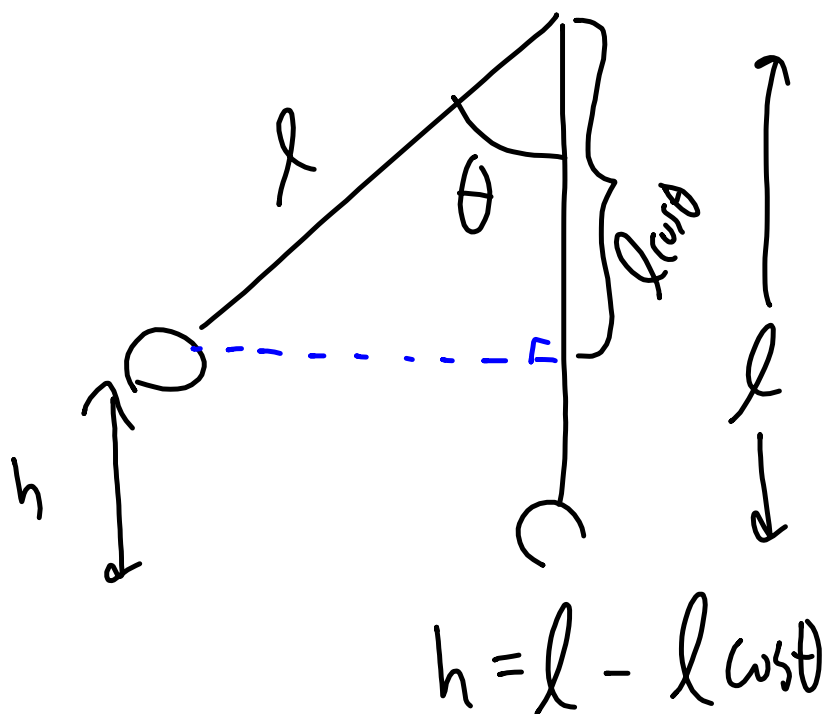












## The many ways to calculate Work

$$W = Fs$$

$$W = Fs \cos\theta$$

$$W = F \text{ (s in the direction of F)}$$

$$W = \text{(F in the direction of s) s}$$

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

$$W = \int F dx$$

Find the work done by the force going from the start position to the end position.

$$F = 2\text{ N } \hat{i} - 5\text{ N } \hat{j}$$

$$\text{start: } 3\text{ m } \hat{i} - 2\text{ m } \hat{j}$$

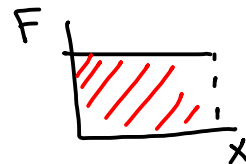
$$\text{end: } 4\text{ m } \hat{i} + 4\text{ m } \hat{j}$$

answer: -28 J

## Non-constant Forces

$$F = -4x$$

Find the work done by the force from  $x = 1$  to  $x = 2$ .

$$W = \vec{F} \cdot \vec{s} = F_x$$


$$W = \int F ds$$

$$= \int F_x dx + \int F_y dy + \int F_z dz$$

$$= \int_1^2 (-4x) dx$$

$$= \left[ \frac{-4x^2}{2} \right]_1^2 = \left[ -2x^2 \right]_1^2$$

$$= \left[ -2(2)^2 - -2(1)^2 \right]$$

$$= -8J + 2J$$

$$\textcircled{-6J}$$

$$F = 6x^2$$

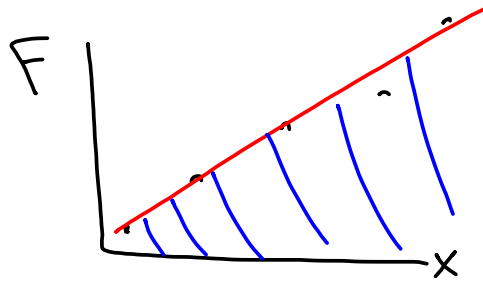
Work done from  $x=0$  to  $x=1$ .

$$W = \int F_x dx$$

$$= \int_0^1 (6x^2) dx$$

$$= \left[ \frac{6x^3}{3} \right]_0^1 = \left[ 2x^3 \right]_0^1$$

$$= \left[ 2(1)^3 - 2(0)^3 \right]$$



$$y = mx + b$$

$$F = -kx \quad F = -2x$$

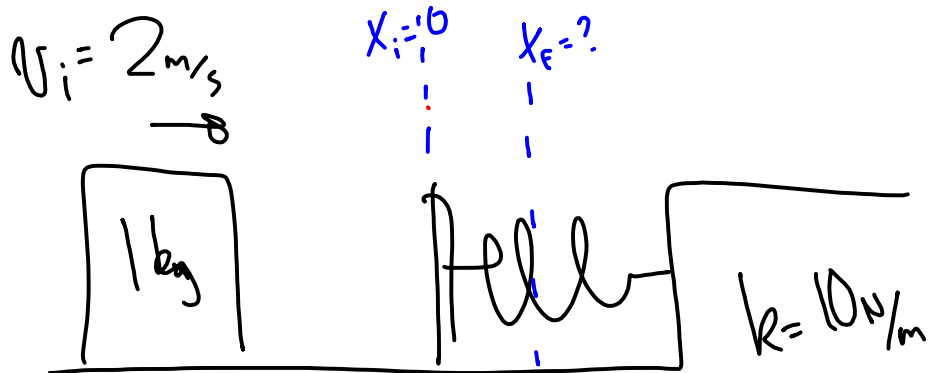
$$W_s = \int_{x_i}^{x_f} F dx$$

$$= \int_{x_i}^{x_f} (-kx) dx$$

$$\left[ -\frac{kx^2}{2} \right]_{x_i}^{x_f}$$

$$-\frac{1}{2}kx_f^2 - \left( -\frac{1}{2}kx_i^2 \right)$$

$$-\frac{1}{2}k(x_f^2 - x_i^2)$$



$$W_{\text{total}} = \Delta K$$

$$W_{\text{total}} = \Delta K$$

$$W_{ng} + W_n + W_s = \Delta K$$

$$= \frac{1}{2} m (v_f^2 - v_i^2)$$

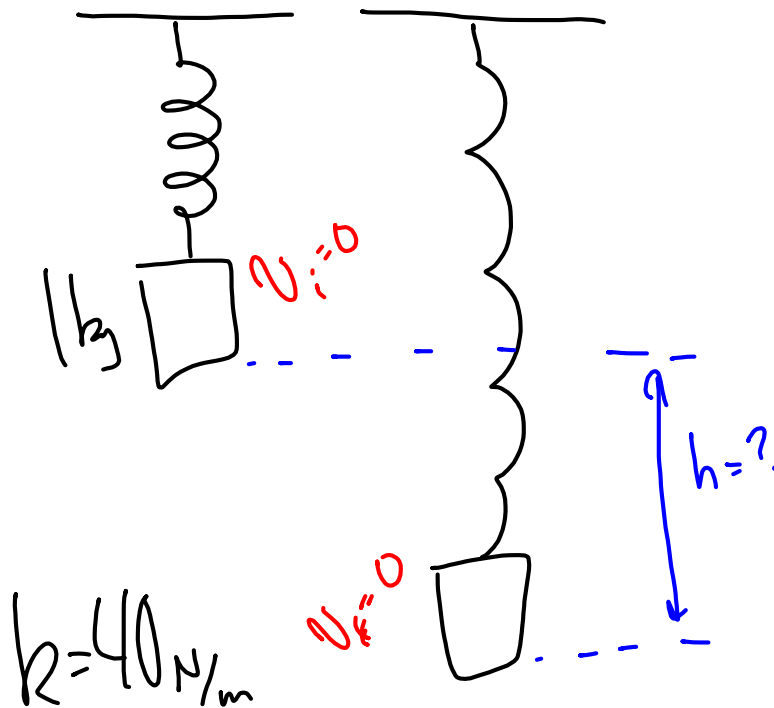
$$= \frac{1}{2} (1) (0^2 - 2^2)$$

$$-\frac{1}{2} k (x_f^2 - x_i^2) = -2 \text{ J}$$

$$-\frac{1}{2} (10) (x_f^2 - 0^2) = -2$$

$$-5 (x_f^2) = -2$$

$$x_f^2 = 0.4 \quad x_f = 0.63 \text{ m}$$



$$W_{mg} + W_s = \Delta K$$

$$+ mgh - \frac{1}{2}k(x_f^2 - x_i^2) = 0$$

$$(1)(10)h - \frac{1}{2}(40)(h^2 - 0) = 0$$

$$10h - 20h^2 = 0$$

$$h(10 - 20h) = 0$$

$$h = 0$$

OR

$$10 - 20h = 0$$

$$h = 0.5 \text{ m}$$



