CONSERVATION OF ENERGY

1. Switching from Work to Energy
2. Conservative & Non-conservative Forces
3. Mechanical Energy, Potentials & Q
4. Energy Thinking
5. Coaster Problems; Choosing h=0
6. U-graphs
7. Practice Problems

Switching from Work to Energy

- If positive work is done on an object, it gains energy.

- If negative work is done on an object, it loses energy.

- Work is the transfer of energy.

ex: If +10 J of work are done on an object, it gains 10 J of energy.

Instead of looking at work done (process view),

look at the energy it had before and after (before/after view)

Conservative Forces

When the object returns to the same position the Mechanical Energy returns to the same value, regardless of path. (The energy only depends on position, regardless of path.)
e.g. springs & gravity

$$F = -kx \quad \cdots \rightarrow \quad W_s = -\frac{1}{2}k(x_f^2 - x_i^2)$$

$$W = -mg \quad \cdots \rightarrow \quad W_g = -mg(y_f - y_i)$$

Conservative Forces

Conservative forces give rise to potential energies (When they do negative work, they store positive energy for you.)

$$U_g = {}^{+}mgh$$

from $h=0$ (your choice)

$$U_e = {}^{+}\frac{1}{2}kx^2$$

from equil pt (not a choice)

Total Mechanical Energy = Potentials + Kinetic

$$E = K + U_g + U_e$$

In General...

The potential energy of a conservative force is the negative integral of force over position.

$$U = -W = -\int F\,dx$$

There's another side to this coin, which we'll see later.

Non-conservative (Dissipative) Forces

When you return to the same position, some Mechanical Energy is lost. They are dissipative because they are path or velocity dependent.

e.g. friction & drag

$\underbrace{\text{friction}}_{\text{kinetic f.}}$ & drag $\rightarrow$ tricky

$$W_f = fd\cos\theta$$

$$= -fd$$

Non-conservative (Dissipative) Forces

When Dissipative forces do negative work, they create positive heat.

$Q = {}^{+}fd$ $f_k = \mu_k N$

thermal or Heat

Energy Thinking

Instead of thinking about forces and motion (during a process) we look at specific points (before and after), and consider the positive energies that the object has at those points.

If there are no dissipative forces, Mechanical Energy is conserved.

If there are dissipative forces, Q must be added to the after.

At each point, ask yourself:

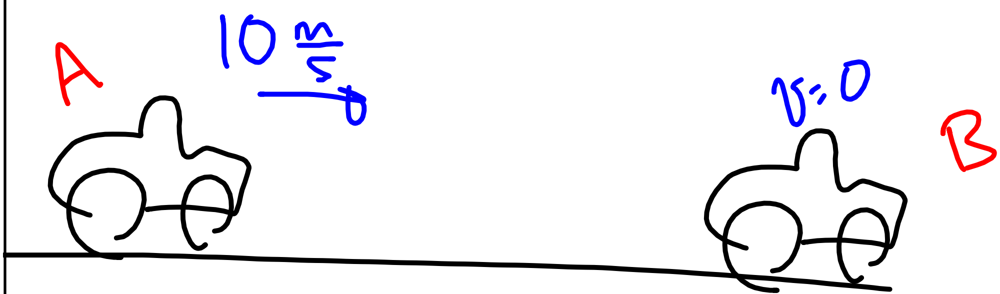
Is the object moving? (K)

Does the object have height? (Ug)

Is a spring compressed or stretched? (Ue)

Has friction been acting? (Q)

Braking Distance



Find the skid distance for a braking car, starting at 30 m/s and ending at rest. The coefficient of friction for asphalt road and tire ranges from 0.7 (dry) to 0.4 (wet).

$$E_A = E_B$$

$$K_A = Q_B$$

$$\frac{1}{2}mv_A^2 = f d_B \qquad f_k = \mu_k N = \mu_k mg$$

$$\frac{1}{2}\cancel{m}v_A^2 = \mu_k \cancel{m} g d_B$$

dry / wet

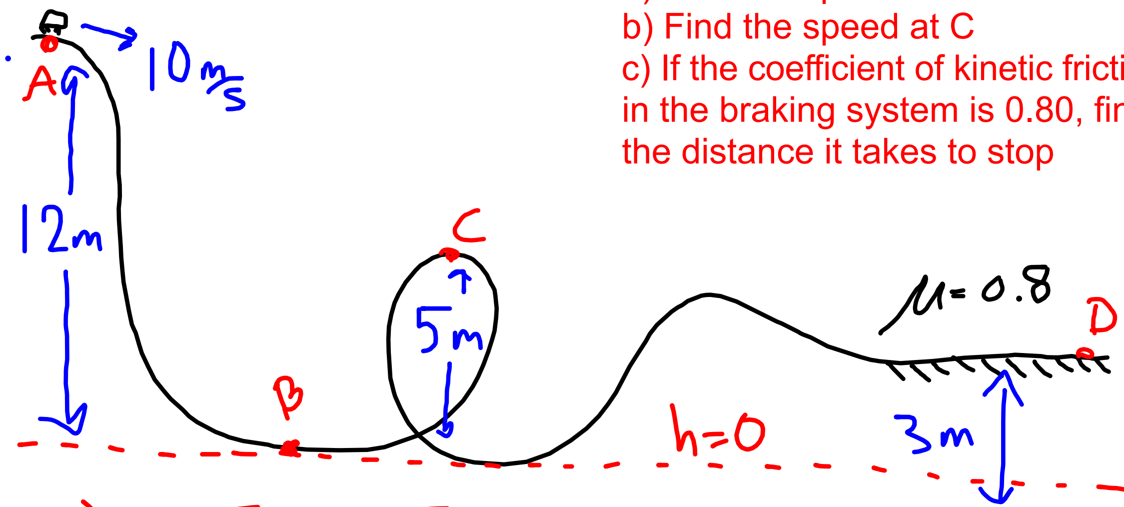
$$\frac{1}{2}(10)^2 = (0.7)(10)d_B \qquad \frac{1}{2}(10)^2 = (0.4)(10)d_B$$

$$7.1\,\text{m} = d_B \qquad 12.5\,\text{m} = d_B$$

roller coaster

The coaster & riders start at A with a velocity of 10 m/s and come to a halt at D.

a) Find the speed at B
b) Find the speed at C
c) If the coefficient of kinetic friction in the braking system is 0.80, find the distance it takes to stop



a) $E_A = E_B$

$K_A + U_{gA} = K_B$

$\frac{1}{2}\cancel{m}v_A^2 + \cancel{m}gh_A = \frac{1}{2}\cancel{m}v_B^2$

$\frac{1}{2}(10)^2 + (10)(12) = \frac{1}{2}v_B^2$

$50 + 120 = \frac{1}{2}v_B^2$

$170 = \frac{1}{2}v_B^2$

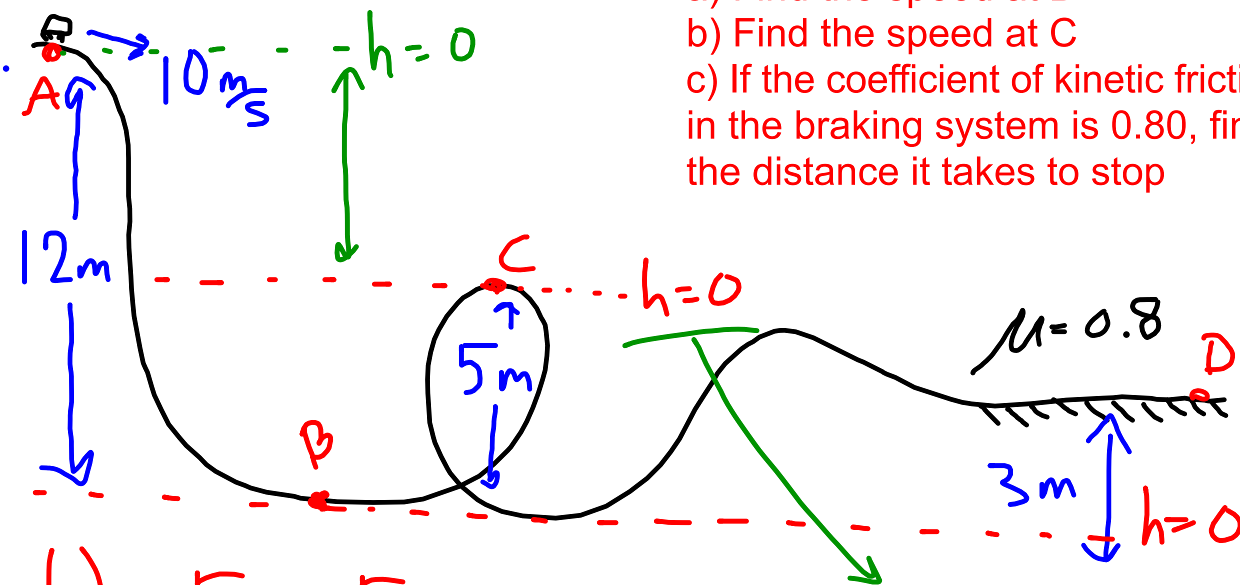
$340 = v_B^2$

$\boxed{18.4\,\frac{m}{s} = v_B}$

roller coaster

The coaster & riders start at A with a velocity of 10 m/s and come to a halt at D.

a) Find the speed at B
b) Find the speed at C
c) If the coefficient of kinetic friction in the braking system is 0.80, find the distance it takes to stop



b) $E_A = E_C$

$\frac{1}{2}\cancel{m}v_A^2 + \cancel{m}gh_A = \frac{1}{2}\cancel{m}v_C^2 + \cancel{m}gh_C$

$\frac{1}{2}(10)^2 + (10)(12) = \frac{1}{2}v_C^2 + (10)(5)$

$50 + 120 = \frac{1}{2}v_C^2 + 50$

$120 = \frac{1}{2}v_C^2$

$240 = v_C^2$

$\boxed{15.5\,\frac{m}{s} = v_C}$

$E_A = E_C$

$\frac{1}{2}\cancel{m}v_A^2 + \cancel{m}gh_A = \frac{1}{2}\cancel{m}v_C^2$

$\frac{1}{2}(10)^2 + (10)(7) = \frac{1}{2}v_C^2$

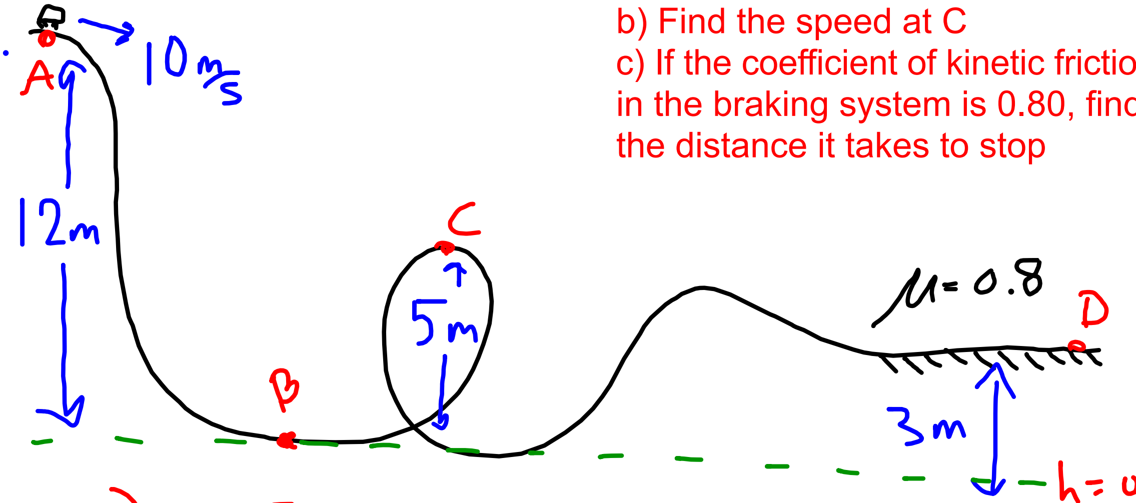
$50 + \underline{70} = \frac{1}{2}v_C^2$

$120 = \frac{1}{2}v_C^2$

roller coaster

The coaster & riders start at A with a velocity of 10 m/s and come to a halt at D.

a) Find the speed at B
b) Find the speed at C
c) If the coefficient of kinetic friction in the braking system is 0.80, find the distance it takes to stop



c) $E_A = E_D + Q_D$

$$\frac{1}{2}mv_A^2 + mgh_A = mgh_D + fd_D \qquad f_k = \mu_k N = \mu_k mg$$

$$\frac{1}{2}\cancel{m}v_A^2 + \cancel{m}gh_A = \cancel{m}gh_D + \mu_k \cancel{m} g d_D$$

$$\frac{1}{2}(10)^2 + (10)(12) = (10)(3) + (0.8)(10)\, d_D$$

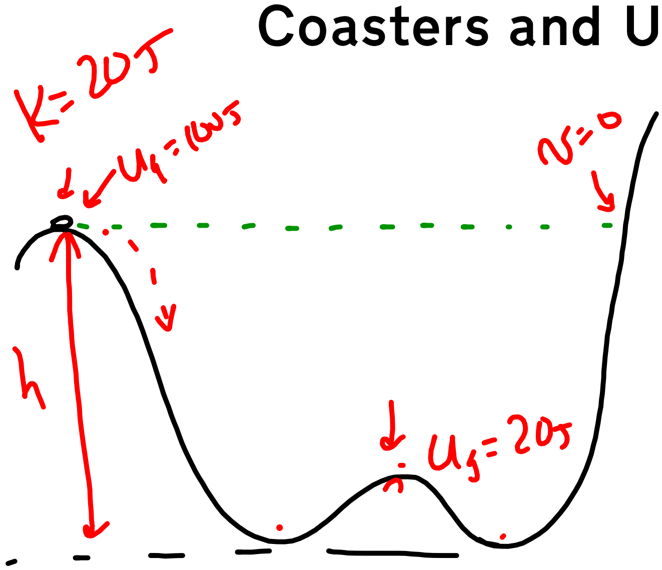
$$50 + 120 = 30 + 8 d_D$$

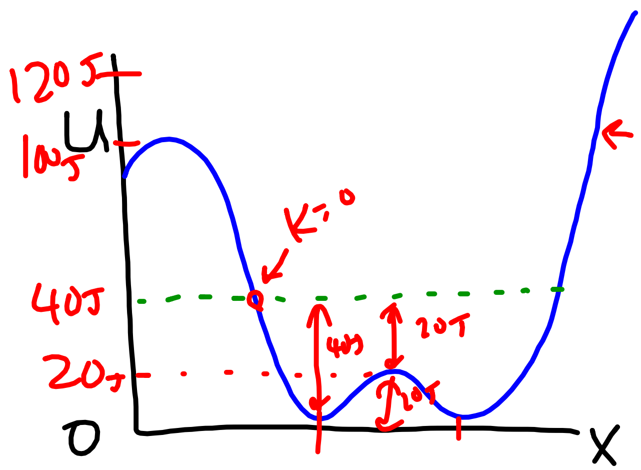
$$170 = 30 + 8 d_D$$

$$\rightarrow 140 = 8 d_D$$

$$\boxed{21.25\,m = d_D}$$

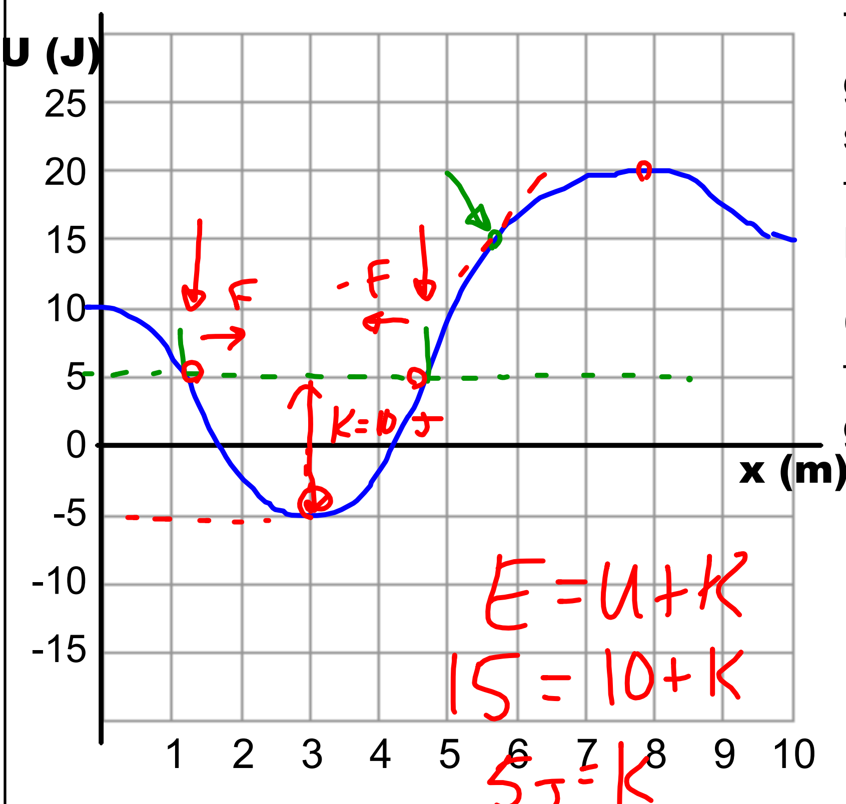
Coasters and U graphs





$$E = U + K$$

$$120_J = 20_J + 100\,J$$



The potential energy graph for an object is shown at left. At x = 3 m, the object has 20 J of Kinetic Energy.

(sub-question: how is force represented on this graph?)

$E = U + K$
$= -5 + 20J$
$= 15J$

In General...

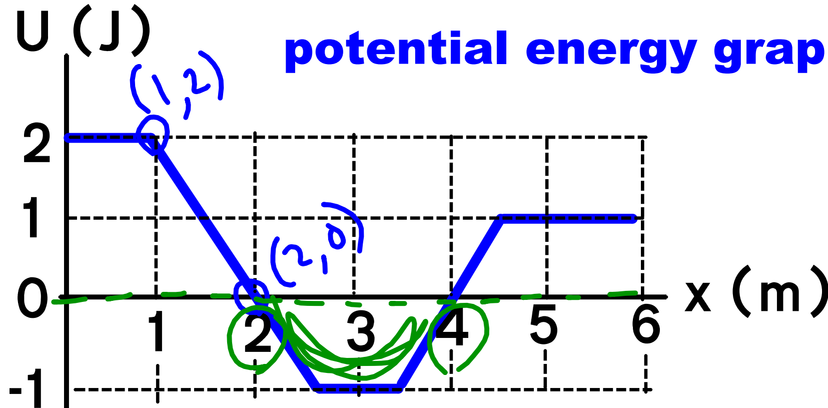
The potential energy is the negative integral of force over position.

$$U = -W = -\int F\,dx$$

The other side of the coin:

$$\boxed{F = -\frac{dU}{dx}} = \boxed{-\text{slope}} \text{ on a U vs position graph}$$

potential energy graphs & forces



This is the potential energy graph for an object. The object's total energy is 1.5 J.

a) Find the kinetic energy at x = 1.5 m
b) Find the kinetic energy at x = 3 m
c) Find the force on the object at x = 2 m
d) If the object's total energy were 0 J, describe the motion it would have.

$E = 1.5\,J$

at $x = 1.5$

a) $E = U + K$

$1.5 = 1 + K$

$\boxed{0.5\,J = K}$

b) at $x = 3\,m$ $U = -1\,J$

$E = U + K$

$1.5\,J = -1\,J + K$

$\boxed{2.5\,J = K}$

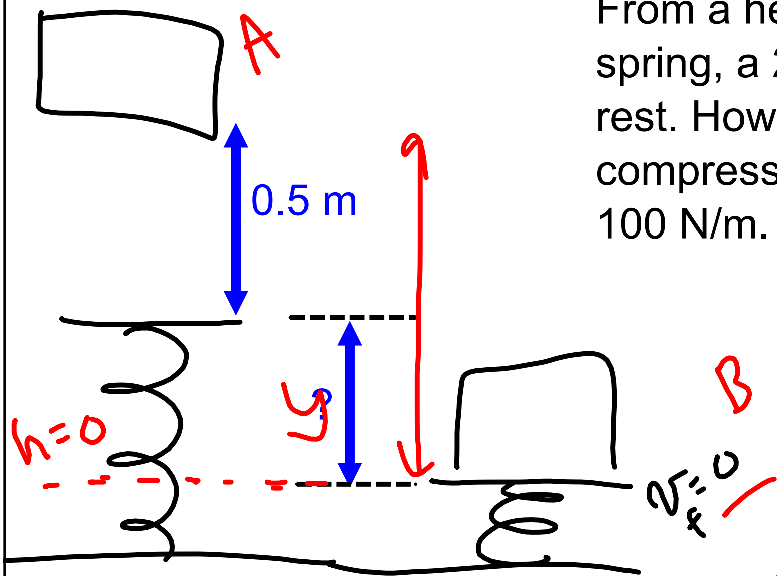
c) $F = -\text{slope}$

$= -\frac{\text{rise}}{\text{run}} = -\frac{0-2}{2-1}$

$= -\frac{-2}{1} = 2\frac{J}{m}$

$\boxed{= 2N}$

Objects Dropped onto Springs



From a height of 0.5 m above a spring, a 2 kg block is dropped from rest. How much will the spring compress? The spring constant is 100 N/m.

$$E_A = E_B$$

$$mgh_A = \frac{1}{2}ky^2$$

$$mg(y+0.5) = \frac{1}{2}ky^2$$

$$(2)(10)(y+0.5) = \frac{1}{2}(100)y^2$$

$$20y + 10 = 50y^2$$

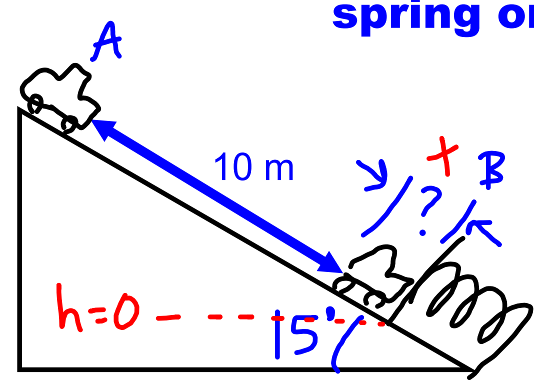
$$0 = 50y - 20y + 10$$

$$0 = 5y - 2y + 1$$

$$y = 0.69 \text{ or } -0.29$$

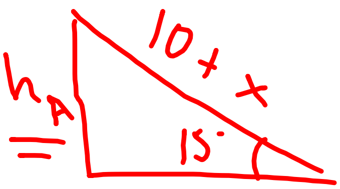
$$\boxed{y = 0.69\,\text{m}}$$

spring on an incline



The 500 kg car's emergency brake fails and it slides down the icy hill. Luckily someone has placed a giant spring at the bottom of the hill (spring constant 10,000 N/m)

How far does it compress the spring?



$$h_A = (10+x)\sin 15^\circ$$

$$E_A = E_B$$

$$mgh_A = \frac{1}{2}kx^2$$

$$mg(10+x)\sin 15^\circ = \frac{1}{2}kx^2$$

$$(500)(10)(10+x)(0.26) = \frac{1}{2}(10{,}000)x^2$$

$$1300(10+x) = 5{,}000x^2$$

$$13{,}0\cancel{00} + 1{,}3\cancel{00}x = 5{,}0\cancel{00}x^2$$

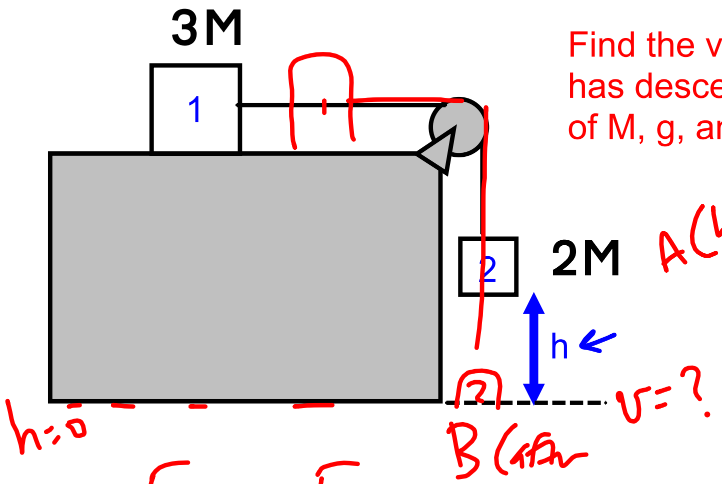
$$0 = 50x^2 - 13x - 130$$

$$x = 1.75 \text{ or } -1.5$$

$$\boxed{x = 1.75\text{ m}}$$

2-Mass System

Find the velocity of Block 2 when it has descended a distance h, in terms of M, g, and h.



$$E_A = E_B$$

$$m_2 g h_A = K_{2B} + K_{1B}$$

$$2\cancel{M}gh = \frac{1}{2}(2\cancel{M})v_B^2 + \frac{1}{2}(3\cancel{M})v_B^2$$

$$2gh = v_B^2 + \frac{3}{2}v_B^2$$

$$2gh = \frac{5}{2}v_B^2$$

$$\frac{4}{5}gh = v_B^2$$

$$v^2 = v_0^2 + 2a\Delta y$$

$$\boxed{\sqrt{\frac{4}{5}gh} = v_B}$$