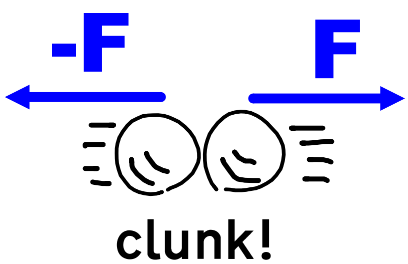
LAW OF CONSERVATION OF MOMENTUM

In an isolated system, the total momentum remains constant.

$$\boxed{\sum F_{ext} = 0} \longrightarrow \boxed{p_i = p_f}$$

NOTE: If the time of a collision is very short, then even if there is an external force like friction, the impulse it exerts will be small and the total momentum will be essentially constant.

Why Momentum is Conserved



In a collision, Forces are equal and opposite (3rd Law)

$$-Ft = Ft$$

Times of contact are the same, so the impulses are also equal and opposite.

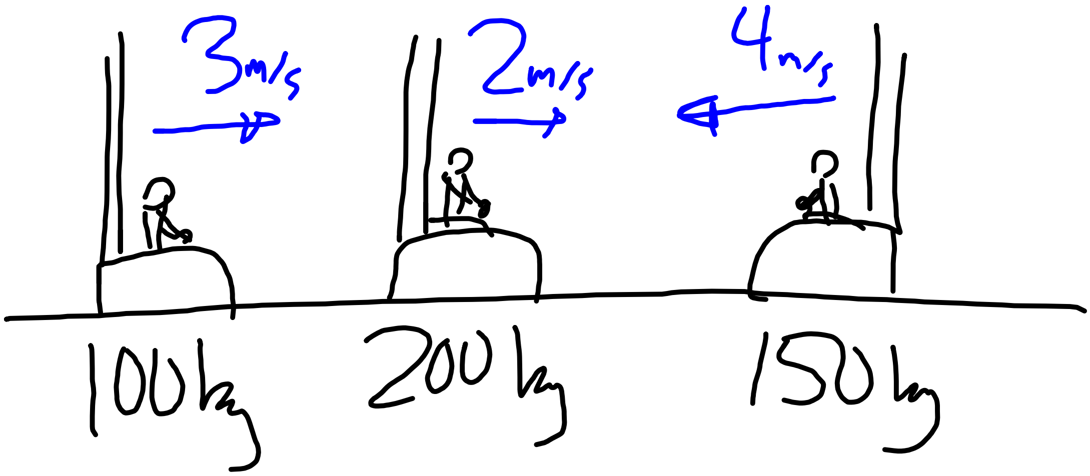
$$-\Delta p = \Delta p$$

Since impulse is equal to the change in momentum, the changes in momentum are also equal and opposite.

That means that the momentum one object gains is the same as the momentum the other object loses.

Therefore the total momentum is constant.

Total Momentum

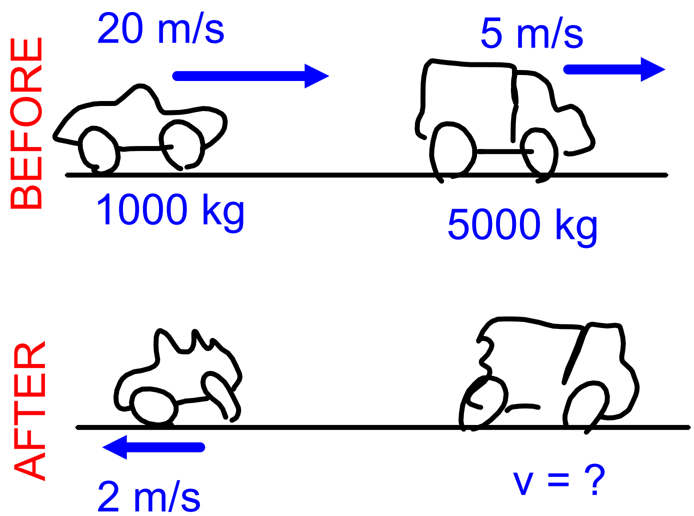


$$p_{tot} = m_1 v_1 + m_2 v_2 + \cdots$$

$$= (100)(3) + (200)(2) + (150)(-4)$$

$$= 300 + 400 - 600$$

$$p_{tot} = 100 \ \frac{kg\,m}{s}$$



Find the velocity of the truck afterward.

$$p_i = p_f$$

$$m_c v_{ci} + m_T v_{Ti} = m_c v_{cf} + m_T v_{Tf}$$

$$(1000)(20) + (5000)(5) = (1000)(-2) + (5000) v_{Tf}$$

$$20{,}000 + 25{,}000 = -2000 + 5000 v_{Tf}$$

$$45{,}000 = -2000 + 5000 v_{Tf}$$

$$\frac{47{,}000}{5000} = v_{Tf}$$

$$\boxed{9.4\ \text{m/s} = v_{Tf}}$$

TYPES OF COLLISIONS

inelastic

totally inelastic

objects stick

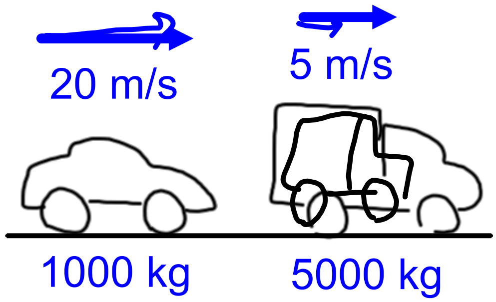
"complete splat"

elastic

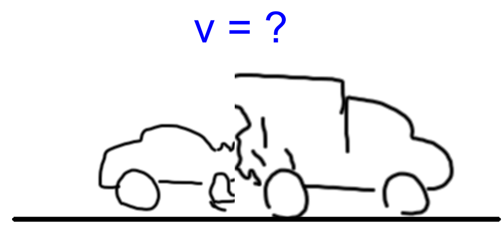
Kinetic E conserved

"the perfect bounce"

Completely Inelastic Collisions



BEFORE



AFTER

$$m_c v_{ci} + m_T v_{Ti} = (m_c + m_T) v_f$$

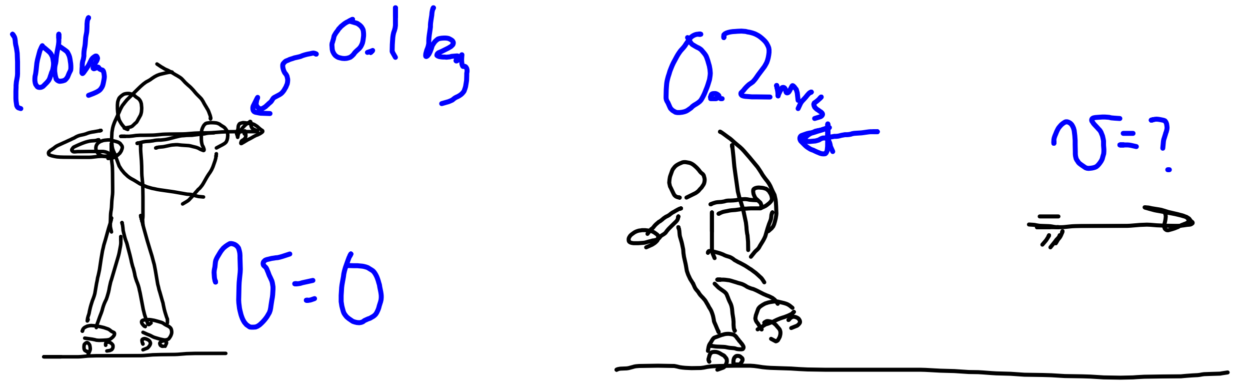
$$(1000)(20) + (5000)(5) = (1000 + 5000) v_f$$

$$45{,}000 = 6000\, v_f$$

$$\boxed{7.5 \tfrac{m}{s} = v_f}$$

{20,000,5,-4} {100,5,905}

The Other Side of the Coin



BEFORE AFTER

$$p_i = p_f$$

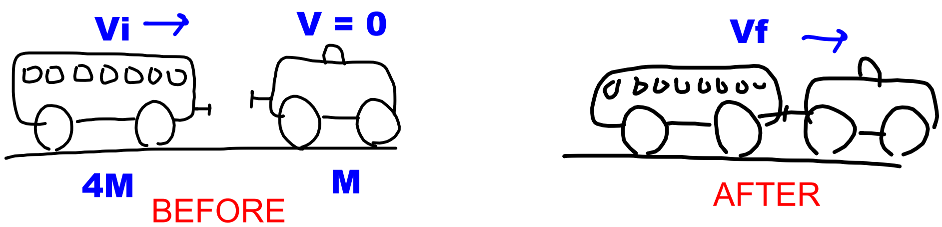
$$(100 + 0.1)(0) = (100)(-0.2) + (0.1)v_f$$

$$0 = -20 + 0.1 v_f$$

{100, 0, -0.2} {0.1, 0, 200}

$$20 = 0.1 v_f$$

$$\boxed{+200\,\frac{m}{s} = v_f}$$



Find the fraction of Kinetic Energy lost in the totally inelastic collision

$\frac{K_f}{K_i}$ = fraction of K conserved

$$\frac{\cancel{\frac{1}{2}}(5\cancel{M})v_f^2}{\cancel{\frac{1}{2}}(4\cancel{M})v_i^2} = \frac{5v_f^2}{4v_i^2}$$

$$p_i = p_f$$

$$(4M)v_i = (5M)v_f$$

$$\frac{4\cancel{M}}{5\cancel{M}}v_i = v_f$$

$$= \frac{5\left(\frac{4}{5}v_i\right)^2}{4v_i^2}$$

$$= \frac{5\left(\frac{16}{25}\cancel{v_i^2}\right)}{4\cancel{v_i^2}}$$

$$= \frac{\cancel{5}\left(\frac{\cancel{16}}{\cancel{25}}\right)\frac{4}{5}}{\cancel{4}}$$

$$= \frac{4}{5} \text{ conserved}$$

$$\boxed{\text{lost: } \frac{1}{5}}$$

Elastic Collisions

Both Momentum and Kinetic Energy are conserved.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

(algebra = yikes!)

Huygen's Relation for Elastic Collisions

Object 1's before and after velocities added together equal object 2's before and after velocities added together.

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

Elastic Collisions

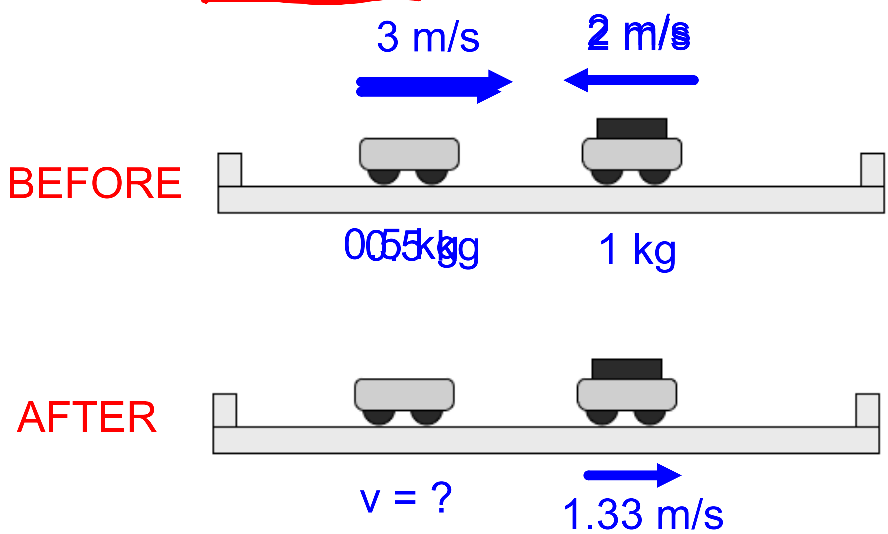
My advice: use Huygen's Relation and Conservation of Momentum.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

(Easier algebra!)

Linear elastic collision - different masses



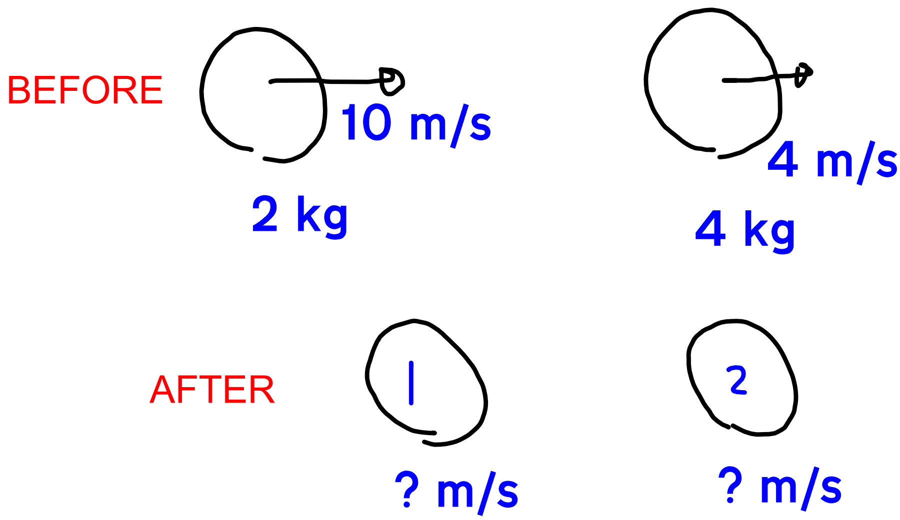
$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$3 + v_{1f} = -2 + 1.33$$

$$3 + v_{1f} = -0.67$$

$$\boxed{v_{1f} = -3.67 \frac{m}{s}}$$

Elastic Collisions



$$p_i = p_f$$

$$(2)(10) + (4)(4) = 2v_1 + 4v_2$$

$$20 + 16 = 2v_1 + 4v_2$$

$$36 = 2v_1 + 4v_2$$

$$18 = v_1 + 2v_2$$

$$10 + v_1 = 4 + v_2$$

$$v_1 = -6 + v_2$$

$$18 = (-6 + v_2) + 2v_2$$

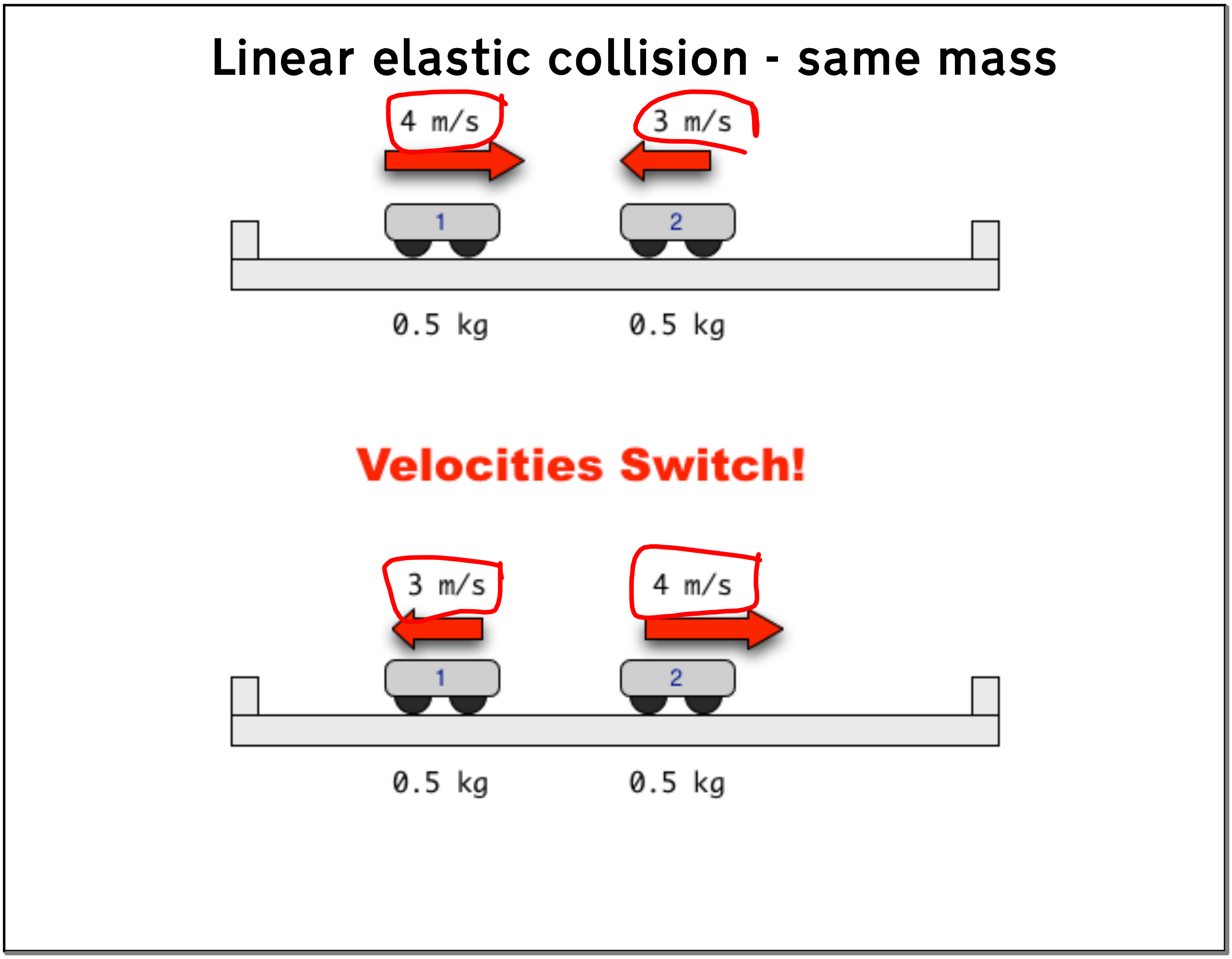
$$18 = -6 + 3v_2$$

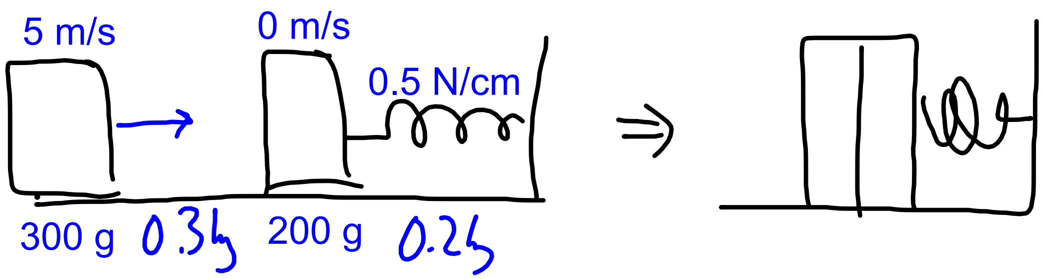
$$24 = 3v_2$$

$$\boxed{8\,\tfrac{m}{s} = v_2}$$

$$v_1 = -6 + (8)$$

$$\boxed{v_1 = 2\,\tfrac{m}{s}}$$


Linear elastic collision - same mass
4 m/s
3 m/s
1
2
0.5 kg
0.5 kg
Velocities Switch!
3 m/s
4 m/s
1
2
0.5 kg
0.5 kg



Assuming the surface has negligible friction, find the maximum compression of the spring.

$$p_i = p_f$$

$$(0.3)(5) + (0.2)(0) = (0.3+0.2)v_f$$

$$1.5 = 0.5v_f$$

$$3\frac{m}{s} = v_f$$

$$E_A = E_B$$

$$K_A = U_{eB}$$

$$k = 0.5\frac{N}{cm}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

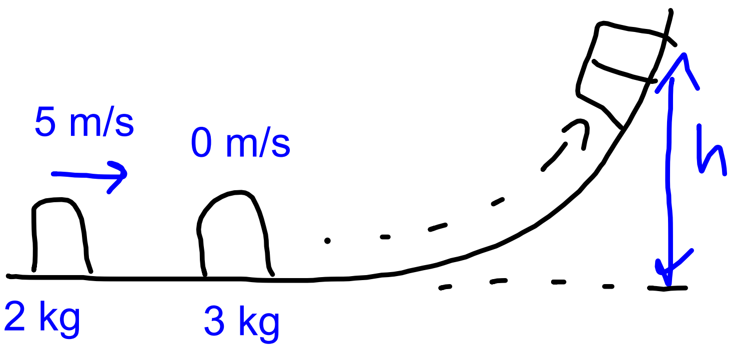
$$k = 50\frac{N}{m}$$

$$\cancel{\frac{1}{2}}(0.5)(3)^2 = \cancel{\frac{1}{2}}(50)x^2$$

$$4.5 = 50x^2$$

$$0.09 = x^2$$

$$\boxed{0.3m = x}$$



The blocks collide and stick. Find the maxiumum height to which they rise afterward. (Assume the surface has negligible friction.)

$$p_i = p_f$$

$$(2)(5)+(3)(0)=(2+3)v_f$$

$$10 = 5v_f$$

$$2\,\tfrac{m}{s} = v_f$$

$$E_A = E_B$$

$$K_A = U_{gB}$$

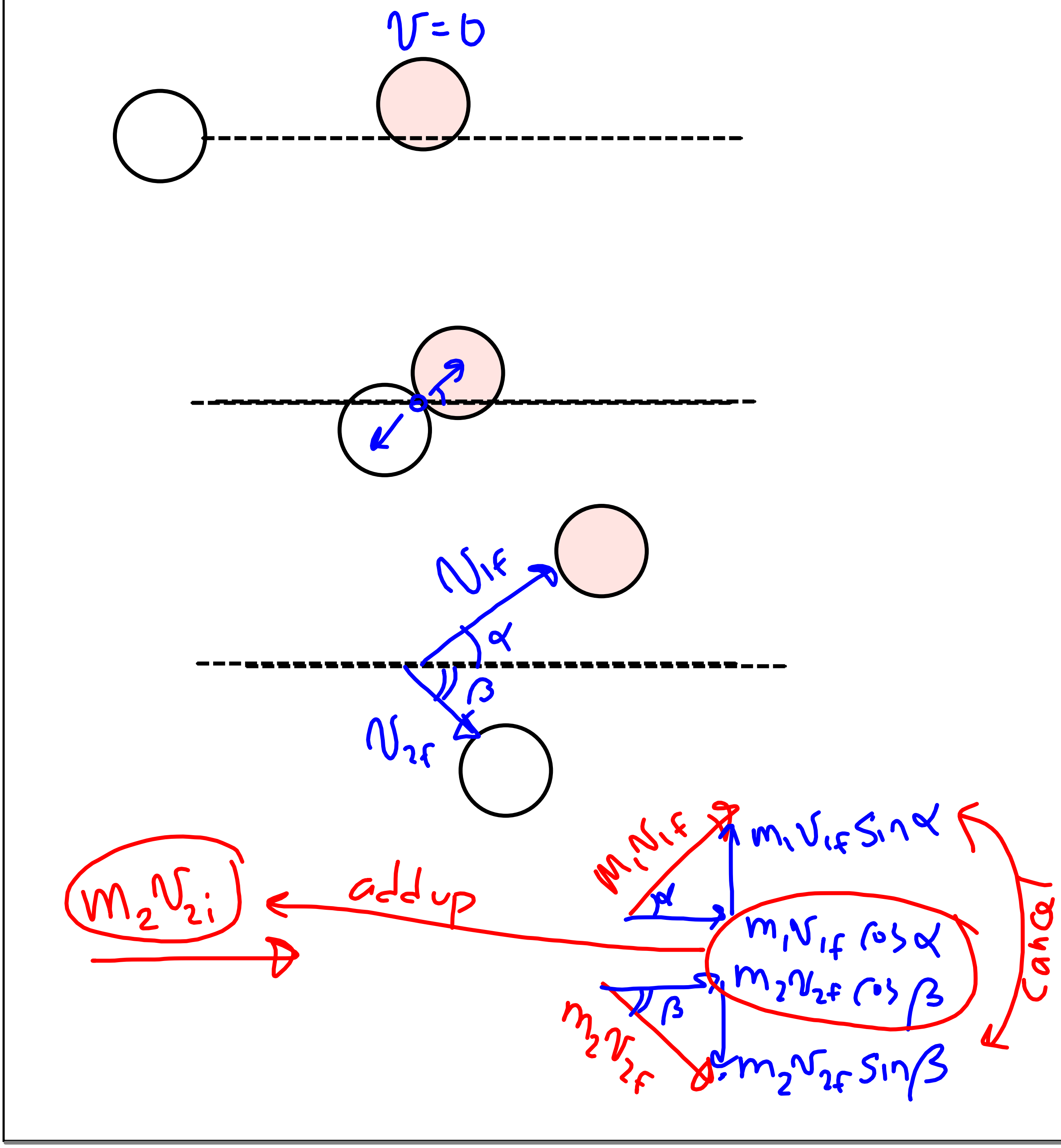
$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

$$\frac{1}{2}(2)^2 = (10)h$$

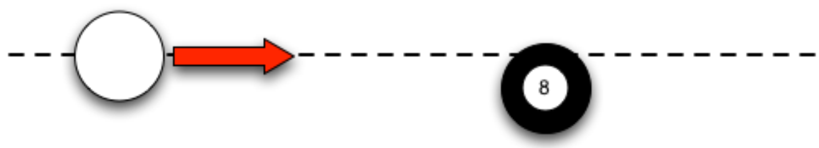
$$2 = 10h$$

$$\boxed{0.2m = h}$$

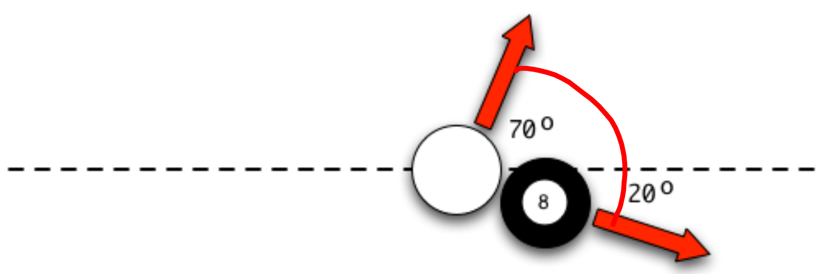
Non-linear collisions



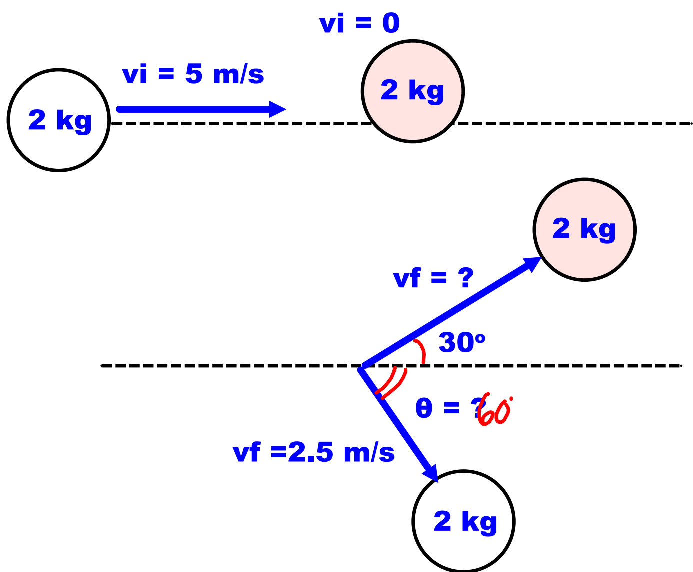
Non-linear elastic collision - same mass

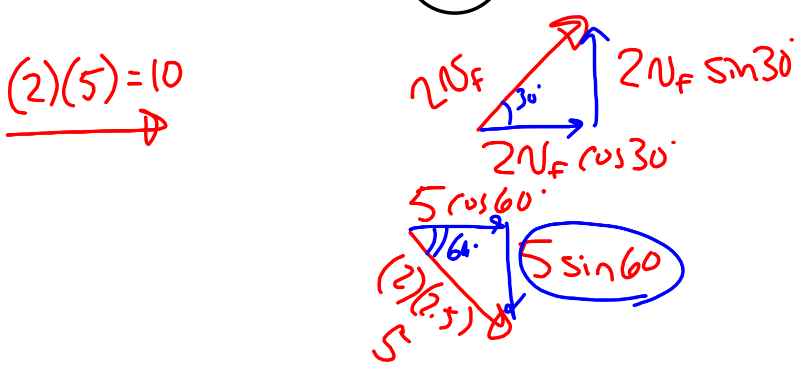


They separate at 90 degrees!



Elastic 2D Collision





$$p_{x_i} = p_{x_f}$$

$$10 = 5\cos 60^\circ + 2v_f \cos 30^\circ$$

$$p_{y_i} = p_{y_f}$$

$$0 = -5\sin 60^\circ + 2v_f \sin 30^\circ$$

$$0 = -4.33 + v_f$$

$$\boxed{v_f = 4.33 \tfrac{m}{s}}$$