Universal Gravitation Guide

Force & Acceleration

$$F = \frac{Gm_1m_2}{r^2}$$

Law of Universal Gravitation. Calculates force of gravity (weight) between two masses. You can't use mg over long distances because g won't be constant.

$$g = \frac{GM}{r^2}$$

Gravitational Field Strength. Calculates the gravitational field strength in N/kg, which is equivalent to the acceleration due to gravity in m/s^2.

$$F = \tfrac{4}{3}\pi G\rho m r$$

Force of Gravity Inside a Solid Planet. ρ is the density of the planet (total Mass divided by total Volume.) Note: inside a shell, the force of gravity cancels out.

Energy

$$U = -\frac{Gm_1m_2}{r}$$

Gravitational Potential Energy. For total potential, calculate this for every pair of masses. You can't use mgh over long distances because g won't be constant. It is not zero on the surface of a planet because r would be measured from the center of the planet. In fact, you have no choice of where to put zero Grav PE because it is zero at an infinite distance. It starts at zero at infinity and decreases to a more and more negative value as you get closer.

$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r}$$

Total Energy. Kinetic E is the same as always, but you can't use mgh (see above.)

$$E = -\frac{GMm}{2r}$$

Total Energy for a Circular Orbit. This is a shortcut derived from the total energy equation with the velocity for a circular orbit substituted in.

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

Escape Velocity. Calculates the velocity required to go an infinite distance away when starting from a distance r from the center of a planet mass M. The mass of the thing escaping is irrelevant. Obtained by setting the total energy equation equal to 0. (You want to run out of Kinetic E at infinity which is also where Grav PE happens to be zero.)

Important: r is the CM to CM distance, not always an actual radius.

Circular Orbits

$$v_{orbit} = \sqrt{\frac{GM}{r}}$$

Velocity Required for Circular Orbit. Note: M is the mass of the thing being orbited; the mass of the orbiter is irrelevant. r is the radius of the orbit. Obtained by setting the force of gravity equal to the centripetal force.

$$v = \frac{2\pi r}{T}$$

Speed for Uniform Circular Motion. This equation works for constant speed around the circle - which is true for circular orbits. T is the period of the orbit.

$$4\pi^2 r^3 = GMT^2$$

Radius Cubed is Proportional to Period Squared. Calculates period of the orbit based on the radius of the orbit and the mass of the thing being orbited. Mass of the orbiter is irrelevant. Obtained by setting the above two equations equal.

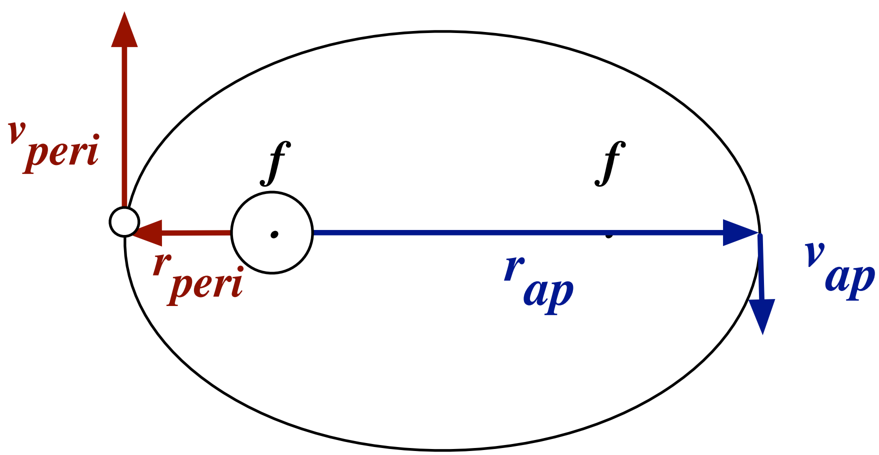
Elliptical Orbits

$$r_{peri} v_{peri} = r_{ap} v_{ap}$$

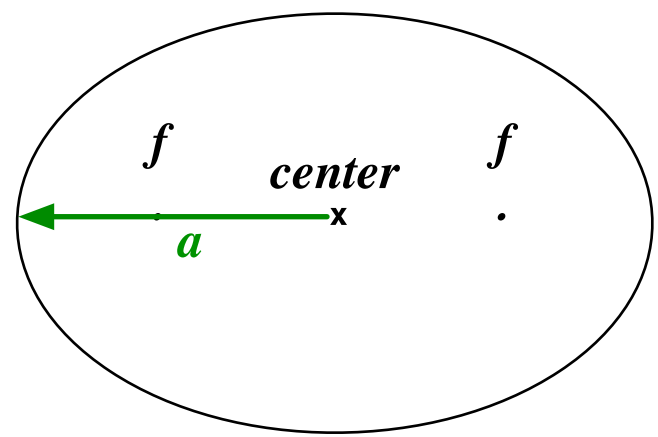
Relationship Between Velocity at Perihelion and Aphelion for Elliptical Orbit. Based on Angular Momentum being constant. "ap" = farthest, "peri" = closest, and "helios" = sun (see diagram.)

$$4\pi^2 a^3 = GMT^2$$

Radius Cubed is Proportional to Period Squared for Elliptical Orbit. Similar to the one for circular orbits, except you should use the a, semimajor axis, rather than r (see diagram.)



Perihelion & Aphelion



Semimajor axis